

Enriched Assessment Styles in Mathematics

Praneetha Singh

*The Hills Grammar School, Kenthurst, New South Wales, Australia
Australian Independent School Mathematics Professional Learning*

Abstract

Assessment has long been a cornerstone in educational reform in Australia, fuelled by meritocracy, accountability and the value of programmatic efforts to improve teaching and learning. For this era, the assessments being used and the explicit policy and practical purposes those assessments are expected to serve are key to eliciting educational reform. The relationship between assessment and the teaching of Mathematics is vital for effective learning. Teaching is usually regarded as conveying knowledge while assessment is simply a means of determining the outcome of the instruction. Essentially what gets taught is tested. Hence student learning and teaching with appropriate assessment are closely linked. It follows that testing affects teaching and teaching affects learning. Learning not only involves the acquisition of knowledge, but being able to use this knowledge in a variety of new situations. The interactions between assessments and classroom learning are becoming increasingly important as we move into an era of student focussed learning rather than teacher-centred learning. Through practical examples, this paper will explore a range of enriched alternate assessment style tasks and the implications for assessment.

1. Introduction

The research objectives were

- To test the effects of using varied enriched assessment styles on students' achievements of educational outcomes and
- To discover whether these diverse assessment styles enhance the teaching and learning of Mathematics.

2. Proposed Methodology

Varying performance-based alternate assessment methods have been administered to students and the results analysed in terms of achievement of teaching and educational goals that have been reached by students. Consideration was given to the effect of alternate assessment styles on overall student achievement compared to the conventional

assessments methods, which included multiple choice style questions and pen and paper assessments. Some of the different performance-based assessment methods included in this study were:

- Portfolio assessments;
- Enquiry approaches - (e.g. open ended questions, investigations, thought experiments, projects)
- Student writing activities and assessments in mathematics.

Assessments were administered to students in a co-educational private senior secondary school. Students with diverse academic abilities in years 7 to 11 were assessed. Testing was done over a period of 12 months. Grading and scoring techniques were also analysed.

3. Why Alternate Assessment Styles?

- Gives ongoing information on strengths and weaknesses of each student. This provides the teacher with an opportunity to work with those weaknesses
- Student who do not have the skills to take formative tests, will have a better chance to succeed in showing their abilities in these assessments, thus building and improving their self esteem
- Enables students to display their knowledge in varying ways
- Allows students to learn at their own pace
- Helps focus on what students do know and can do rather than what they don't know

4. Results and Discussion

Teachers were asked "Why do we assess students?" Responses fell broadly into the following categories:

- To find out what my students know and can do;
- To help me know what to teach next;
- To measure the effectiveness of my teaching;

- To provide feedback to students on their learning progress;
- To inform parents and other staff of academic progress
- To help students (year 11 and 12) prepare for their Higher School Certificate (this is the final external examination students sit in year 12)
- Because my Head of Department/principal/community expects it.

At face value these reasons seem plausible. The first four imply some form of subsequent action, but this is seldom followed through. It is a well-known fact that assessment without subsequent action is unlikely to lead to improved learning. Hence the *action* that follows the *act* of assessment is potentially more powerful. Having stated that, the actual style in which students are assessed, plays a vital role, too, in the teaching and learning process.

In the 1980's, in Australia, an increasing consensus emerged among classroom teachers that traditional forms (pen and paper and multiple choice style) of assessment proved insufficient in meeting all the revised goals which teachers held for assessment. The concept of valuing genuine student understanding, problem solving and group skills (including communicating effectively in mathematics) and importantly for students to use their knowledge in "real life situations", resulted in expanding the repertoire of assessment techniques from the conventional methods to more "informal", enriched methods. It is through "valid and proper assessment we communicate most clearly to students those activities and learning outcomes we value" [1].

In the 1990's issues of classroom, school and system level accountability enforced greater emphasis on proposing desired outcomes and ways of collecting, documenting and reporting student growth over a period of time. Thus, the challenge of ensuring assessments is meaningful yet manageable arose.

Therefore the most effective teaching and subsequent academic outcomes would be achieved by exposing students to varying styles of assessment rather than solely the conventional methods.

Good questioning techniques in everyday teaching also enrich the learning process and assist students in preparation for assessments. It is a fundamental tool of effective teachers. Unfortunately research shows not many teachers fall into this category [2]. The right type of questioning helps stimulate the mathematical thinking that is necessary for meaningful engagement in open-ended problems and investigations. Teachers should develop a questioning style that guides, supports and stimulates their students without removing the responsibility for problem-solving process from their students.

Examples of questions within the context of open-ended mathematical tasks can be broadly placed into four main categories [3]. These questions can be useful to the teacher too, as besides helping guide the students through investigations whilst stimulating their mathematical thinking, teachers can gather information about their students' knowledge and strategies.

4.1. Start off questions

These are open-ended questions that steers the students' thinking in a general direction and gives them a starting point. Examples: How could you sort these..? How many ways can you find to ... ? What happens when we ... ?

4.2. Stimulation of mathematical thinking

This enhances strategies that assist students to see patterns and relationships, which aids the formation of a strong conceptual network. The questions can serve as a prompt when students become 'stuck'. A danger is that teachers may be tempted to turn these questions into instructions, which is far less likely to stimulate thinking and removes responsibility of investigation from the student. Examples: What is the same/ different? Can you see a pattern? How can this pattern help you find an answer? What do think comes next? Why? Is there a way to record what you've found that might help us see more patterns? What would happen if....?

4.3. Explanation questions

These questions prompt students to explain what they are doing or how they arrived at a solution. They allow the teacher to see how the students are thinking, what they understand and what level they are operating at. Students need to be given some time to think about their solutions and methodology before these questions can be asked. Examples: What have you discovered? How did you find that out? Why do you think that? What made you decide to do it that way?

4.4. Class discussion questions

These questions help guide the efforts of the class and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realisation of mathematical ideas and relationships. It encourages children to evaluate their work. Examples: Who has the same answer/ pattern/ grouping as this? Who has a different solution? Are everybody's results the same? Why/why not? Have we found all the possibilities? How do we know? Have you thought

of another way this could be done? Do you think we have found the best solution?

This method can be combined with Bloom's Taxonomy [4] of the six thinking levels:

- *Memory*: The student recalls or memorises information
- *Translation*: The student changes information into a different symbolic form or language
- *Interpretation*: The student discovers relationships among facts, generalisations, definitions, values and skills
- *Application*: The student solves a life-like problem that requires identification of the issue and selection and use of appropriate generalisations and skills.
- *Analysis*: The student solves a problem in the light of conscious knowledge of the parts of the form of thinking.
- *Synthesis*: The student solves a problem that requires original, creative thinking
- *Evaluation*: The student makes a judgement of good or bad, right or wrong, according to the standards he values.

These two methods of categorising types of questions overlap and support each other. For example, the questions: Can you see a pattern? How can this pattern help you find an answer? This relates to Interpretation. Hence asking the right questions can lead to the prompting and deep thought processes that are required by students to enhance their assessment techniques.

Observations of students' performances in conventional formative assessment practices were reviewed and the results indicated a few key weaknesses as set out below:

- Classroom evaluation practices generally encouraged superficial and rote learning, concentrating on recall of isolated details, usually areas of knowledge which students soon forgot;
- The review of the assessment is not thorough, very little reflection and critical discussions take place;
- The grading element is over-emphasised whilst the learning element is under-emphasised;
- There is a tendency to use a normative rather than a criterion approach, which results in competition between students rather than personal improvement of each. The evidence is that with such practices the effect of feedback is to suggest to weaker students that they lack ability. This demotivates them and lowers their self-confidence in their capacity to learn
- They permit comparisons among students and fulfil accountability demands. They certify students for entry into other educational programs or institutions, but they do not provide

any specific information about what the student has achieved.

Recent research has confirmed this general picture [5]; [6]; [7]; [8]

Like teaching and learning, assessment is not a singular entity. It is complex and dynamic and it deserves to be differentiated and understood in all of its intricacy. Educators will therefore need to think about the various purposes for assessment and choose the purposes they believe are important and how to realise these purposes every day in their classrooms.

Thus, this style of assessing does not really tell teachers what they need to know about their students' learning; hence the need for varying performance-based alternate assessment methods rose.

A snapshot of some of the different assessment styles are discussed below

5. Portfolio Assessments

Portfolio assessments could fall broadly into three categories:

- *The Showcase*: This type of portfolio focuses on the student's best and most representative work. A variety of work is selected to reflect breadth of talent. Therefore, in this portfolio the student selects what he or she thinks is representative of his or her best work. This folder is most often seen at open classrooms and/or parent visitations.
- *Teacher-Student Portfolio*: This type of portfolio is often called the "working portfolio" or a "working folder". This is an interactive teacher student portfolio that aids in communication between teacher and student. The teacher and student have discussions about what to add or delete within the content of the portfolio.
- *Teacher Alternative Assessment Portfolio*: All the items in this type of portfolio are scored, rated, ranked, or evaluated. Teachers can keep individual student portfolios that are solely for the teacher's use as an assessment tool. This is a focused type of portfolio and is a model of the holistic approach to assessment.

Teacher-Student portfolios can be regarded as educational portfolio in which a group of projects and papers are stored in a file/folder or electronically. It includes teachers' evaluations and student self-reflections. According to the Northwest Evaluation Association [9], a portfolio is "a purposeful collection of student work that exhibits the student's efforts, progress, and achievements. The collection must include student participation in selecting contents, the criteria for selection, the criteria for judging merit, and evidence of student self-reflection" [10].

A teacher who is interested in documenting a student's improvements in mathematics throughout a

school year can have the student keep a developmental portfolio which doubles up as an alternative assessment portfolio as it contains samples of the student's work along with self-evaluations of specific assignments. Such a portfolio provides specific documentation which can be used for student evaluations and parent conferences. It serves as a great benefit to teachers as they may use an existing portfolio system in order to receive information about an incoming class of students. As a result the teacher gains a better understanding of the ability levels of his or her students prior to the start of the school year. This enables the teacher to plan accordingly.

The Showcase portfolios are designed to represent the student's best work, whilst an alternative assessment portfolio is designed to show how the student's work has evolved overtime, and are comprehensive repositories for the entire student's work. Since current technology allows for the capture and storage of information in the form of text, graphics, sound, and video, students can save writing samples, solutions to mathematics problems and multimedia presentations in one coherent document. Thus digital portfolios tend to be more efficient and many students took great pride in presenting their portfolios to the class.

The implementation of computer-based portfolios for student assessment is therefore an exciting educational innovation. This method of assessment not only offers an authentic demonstration of accomplishments, but also allows students to take responsibility for the work they have done. In turn, this motivates them to accomplish more in the future. A computer-based portfolio system offers many advantages for both the educational and the business communities and should continue to be a popular assessment tool in the "information age."

Thus some of the main goals of portfolios are to see student thinking, student's growth over time, mathematical connections, student views of themselves as mathematicians, and how students' progress in their problem solving skills over a period of time.

6. Enquiry Approaches-(e.g. open-ended questions, investigations, thought experiments)

Enquiry approaches are advocated in mathematics because they engage students and suit a range of student capabilities. They also provide authentic opportunities for teamwork and co-construction of knowledge. Hence, although core mathematical knowledge is essential in an enquiry approach it is not sufficient. As a result, enquiry approaches are strongly encouraged in mathematics education to provide opportunities for a diversity of learners to engage in interesting and challenging tasks [11]. However, this approach requires substantial changes to teaching and assessment [12].

The appropriate level of challenge is important to provide adequate opportunities for learning and high level thinking and reasoning [11]. Challenge and enquiry are interrelated because the cognitive value of a task exists in the opportunity that it provides for students to explore and solve a problem [13]. The opportunity for exploration and problem solving emulates the work of mathematicians and is evident in various forms of enquiry tasks including open-ended tasks, mathematical investigations [14], and thought experiments.

Open-ended refers to a question or problem which has more than one correct answer and more than one strategy to obtain this answer. Open-response refers to a question or problem that may only have one correct answer or one strategy to obtain the answer. In both open-ended and open-response mathematics problems, students are expected to explain or justify their answers and/or strategies. Open-ended questions should challenge students, support concept and skill development, encourage creativity and cater for students with varying mathematical competencies.

The problems are not intended to be ones that can be solved quickly or without thought. However, the challenge provided by these questions should elicit classroom discussion about strategies that may or may not be obvious to the average student.

An example of such a problem is given below.

How would you measure how much water is in a puddle? This question was given to year 7 students.

Table 1. Low and exceptional levels of responses to the Puddles task.

Low level response	High level response
Put a ruler through it	Stick a ruler in the puddle, unstick it, then read off how deep it is
Stand in it	Get a ruler, put it horizontally and see how wide it is
Put your hand in it until you find the deepest point	You can scoop out all the water into a jug and see how many litres it is
Walk through it	Pour puddle into a jug, weigh it, then pour water out, weigh jug and subtract to find out weight of water

A year 11 student asked why he achieved a B for an assignment when he thought he answered the task reasonably well. The teacher's response was "you did what you were asked to do. If you want a better grade, then you need to do more". Clear communication and expectations were lacking. It is vitally important to share the expectations and a rubric with the students for such assessments. Students need to understand what they need to know and be able to do, how this will be assessed and the consequences of the assessment early in the process. Open-ended tasks can be assessed using a rubric. The essential features of rubrics are evaluative

criteria, which are used to distinguish between acceptable and unacceptable responses on particular criteria; quality definitions, which describe performance at various levels; and a scoring criterion [15]. It would be beneficial to involve students in the development of rubrics as it may motivate them to create the guidelines used to score their performances. Using rubrics both outlines the expectations for students and eliminates subjective grading practices. They allow teachers to accommodate heterogeneous classes. Rubrics provide an easy way to explain student evaluation to parents.

Table 2. Rubric for assessing Enquiry based tasks

*Performance Levels***Low Response**

Measurement methods proposed are extremely simple or are unclear or unworkable. Explanation is limited and may reveal basic misconceptions about measurement

Medium Response

Measurement methods are fairly clearly presented; they tend to be the most obvious choices. Explanations are adequate, with limited details.

High Response

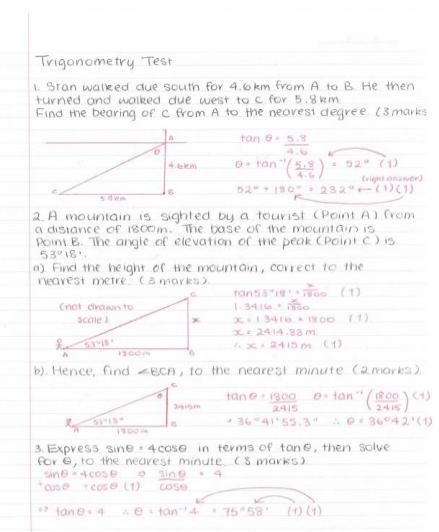
The measurements and tools are clearly presented and may include some original ideas. The response shows a good understanding of the problem. Explanations are clear and include some detail.

Exceptional Response

The methods for measuring go beyond the most obvious to include some strategies that show original thought or sophisticated thinking for the grade level. The response reveals an ability to think through a complex problem. Explanations are clear and effective and include relevant details.

7. Students writing activities and assessments

Year 10 students were placed in groups of 2, given a topic that was recently taught to them and asked to come up with 2 well thought out questions. They were given guidance and a sample question. There were prior discussions in class about the levels of difficulty, grading of the questions (could be sub questions) and a detailed marking rubric. These assessment tasks were then administered to peers and were marked by the students that set the tasks. This proved very useful as attention to detail in responses was highlighted. This led to further class discussion on the understanding of concepts taught and tested. Students understood and appreciated the intensity and time taken to set a meaningful and valid assessment task. Most students answered the questions in this topic very well in the final examination, indicating there was a deep understanding of the topic. Samples of the students test are given below.



Trigonometry Test

1. Stan walked due south for 4.6 km from A to B. He then turned and walked due west to C for 5.8 km. Find the bearing of C from A to the nearest degree. (3 marks)

2. A mountain is sighted by a tourist (Point A) from a distance of 1800m. The base of the mountain is Point B. The angle of elevation of the peak (Point C) is $53^\circ 18'$.

- a) Find the height of the mountain, correct to the nearest metre. (3 marks)

- b) Hence, find $\angle BCA$, to the nearest minute. (2 marks)

3. Express $\sin \theta = 4\cos \theta$ in terms of $\tan \theta$, then solve for θ , to the nearest minute. (3 marks)

4. Find the values of the following in exact form: (4 marks)

a) $\sin(1200^\circ)$

$$= 0.8660254038 \times \frac{\sqrt{3}}{2}$$

b) $\tan(1395^\circ)$

$$= -1 \quad \checkmark$$

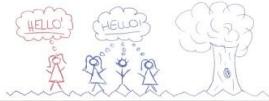
c) $\cos(-405^\circ)$

$$= 0.7071067812 \times \frac{\sqrt{3}}{2}$$

d) $\tan(600^\circ)$

$$= 1.732050808 \times \sqrt{3}$$

(soz if I did all of these wrong) 



Kyna & Ashleigh: markers

Holly

9/15

53%

Year 10 Mathematics Honours Topic Test – Trigonometry and Angles of Magnitude

1. Stan walked due south for 4.6 km from A to B. He then turned and walked due west to C for 5.8 km. Find the bearing of C from A to the nearest degree. (3 marks)

2. A mountain is sighted by a tourist (Point A) from a distance of 1800m. The base of the mountain is Point B. The angle of elevation of the peak (Point C) is $53^\circ 18'$. (3 marks)

a) Find the height of the mountain, correct to the nearest metre.

$$\begin{aligned} \tan \theta &= \frac{BC}{AB} \\ \theta &= \tan^{-1}\left(\frac{5.8}{4.6}\right) \checkmark \\ &= 51^\circ 34' + 180^\circ \checkmark \\ &\approx 231^\circ \checkmark \end{aligned}$$

- b) Hence, find angle BCA , to the nearest minute. (1 mark)

$$\begin{aligned} \tan \theta &= \frac{1800}{2408} \quad \text{ECF} \quad 1/1 \\ \theta &= \tan^{-1}\left(\frac{1800}{2408}\right) \quad 1/1 \\ &\approx 36^\circ 47' \end{aligned}$$

- c) Express $\sin \theta = 4\cos \theta$ in terms of $\tan \theta$, then solve for θ , correct to the nearest minute. (4 marks)

$$\begin{aligned} \sin \theta &= 4\cos \theta \quad (\text{I honestly have no idea if this is right } \checkmark) \\ \frac{1}{\tan \theta} &= 4 \quad (\text{I honestly have no idea if this is right } \checkmark) \\ \theta &= \tan^{-1}\left(\frac{1}{4}\right) \quad 1/4 \\ &= 14^\circ 21' \end{aligned}$$

NAME: _____

SCORE: _____

By Jacklyn + Stephen

1. Inc was going to the beach and decided to get a tan because he saw a sign permitting that he could do so. This sign was 11.3 metres tall and Inc was tanning 69.7 metres away from the base of the sign.

- a) Draw the diagram

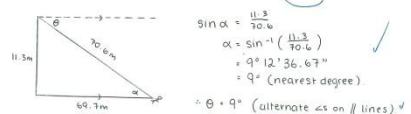
1 mark

- b) Find the distance from where Inc is tanning to the top of the sign (1 d.p)

2 marks

c) Find the angle of depression of the spot where Ince is tanning from the top of the sign (nearest degree) 2 marks

c) Find the angle of depression of the spot where Ince is tanning from the top of the sign (nearest degree) 2 marks



2. Prove that:

$$\frac{\cos A \times \tan A}{\sin A} = 1$$

3 marks

2. Prove that:

$$\frac{\cos A \times \tan A}{\sin A} = 1$$

2 marks (2)

$$\frac{\cos A \times \tan A}{\sin A} = 1 \quad \checkmark$$

$$\frac{\cos A \times \tan A}{\sin A} = \frac{\sin A}{\sin A}$$

$$\therefore \cos A = \tan A$$

$$= 1$$

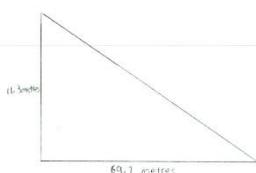
*Length - the length was really good, I finished before the time was up but I didn't finish too quickly either
 *Fairness - all questions were in the topic we studied, except for question 1a) which I thought was too simple and fell into the geometry chapter
 *marking - it was very fair but an explanation as to why I lost marks would have been good to have

SOLUTIONS: JASDEEP & STEPHEN

1. Ince was going to the beach and decided to get a tan because he saw a sign permitting that he could do so. This sign was 11.3 metres tall and Ince was tanning 69.7 metres away from the base of the sign.

a) Draw the diagram

1 mark



b) Find the distance from where Ince is tanning to the top of the sign (1 d.p)

$$x = \sqrt{69.7^2 + 11.3^2} \quad (\text{Pyth})$$

$$= 70.6 \text{ metres}$$

2 marks

Student self-assessment can provide a great degree of ownership of the assessment process to the students and also provides a benefit to both teachers and students as students begin to internalise performance standards. These student-constructed tests help students assess their own growth in knowledge and skills. Students create assessment tasks that they believe would assess objectively the key ideas in the topic(s) under study. This has proven to be excellent revision, and is particularly well received when the teacher makes a commitment to use at least some of the student created questions in a formal assessment.

8. Methods of Assessment influence methods of instruction

A student will learn best if taught in a method deemed appropriate for the student's learning style. Teachers play an essential role in teaching through problem solving. The National Council of Teachers of Mathematics, NCTM (2000) indicated that teachers must "decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" [16] found that teachers in problem-solving classrooms used fewer problems, spent more time on each problem, and asked more conceptual questions than teachers in more traditional classrooms. Despite the importance of the teacher's role in problem-solving instruction, little research suggests how teachers learn to teach through problem solving [17]. For example, instruction in alternative solutions is rarely observed in

Markers - Stephen + Jasdeep

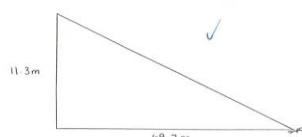
NAME: Kyna

SCORE:

1. Ince was going to the beach and decided to get a tan because he saw a sign permitting that he could do so. This sign was 11.3 metres tall and Ince was tanning 69.7 metres away from the base of the sign.

a) Draw the diagram

1 mark



b) Find the distance from where Ince is tanning to the top of the sign (1 d.p.)

$$11.3^2 + 69.7^2 = 4925.78 \quad \checkmark$$

$$\sqrt{4925.78} = 70.61 \quad \checkmark$$

2 marks

mathematics classrooms, and little is known about how to support more frequent use of alternative solutions by mathematics teachers in their classroom instruction [18].

Some other varying assessment styles that teachers could investigate are:

- Performance tasks - hands-on activities that require students' to demonstrate their ability to perform certain actions. This category of assessment covers an extremely wide range of behaviours', including designing products or experiments, gathering information, tabulating and analysing data and interpreting results.
- Solutions to problems are given to the students; they then design a possible question to fit that solution. This method requires students to design an appropriate question for the detailed set of solutions given to them. This encourages 'reverse' thinking skills and requires extensive thinking.

9. Conclusion

Varying performance-based alternate assessment tasks present assessment challenges for students and teachers. In an enquiry approach, students need to take responsibility for monitoring and critiquing their thinking and the thinking of peers. They also need to be able to explain their ideas and justify their critiques. Thus, even from an early age, students need to be challenged and supported to assume some responsibility for their role in assessments.

There are some advantages as well for example the portfolio assessment might help students see their strengths and weaknesses so that the students are more able to link successes and failures to performance and may also facilitate goal setting. They allow the integration of learning and assessment. Learning based on portfolio assessment can be more student directed, and since evaluation is not based on single scores, instruction based on learning styles is more easily evaluated. The portfolio assessment including open-ended questions can aid teachers in observing how students process mathematics information and also help differentiate the skill levels of individual students but for this to work efficiently; the teaching styles of teachers would have to change.

Another great advantage of alternate assessment is that it empowers students to become involved and decision makers in their learning, thus becoming an informative source for teachers and a learning tool for the students. Alternate assessments capture the significant outcomes we want students to achieve and better match the kinds of tasks they will need to accomplish in order to ensure their future success. Hence the way students are assessed fundamentally affects their learning.

Teachers then need to be constantly reviewing and improving the tasks set and providing constant, relevant and enriched feedback thus enhancing the learning process. They can break away from the monotonous everyday teaching routine as they can organise activities for students that create a pleasant and motivating atmosphere in the classroom, which revives the interest of the student for the subject. The positive benefits of alternate assessments lie not only in its implementation but also in the teachers ability to extend and enrich the curriculum through activities he / she initiates. Thus thorough planning and understanding of the skills students must develop are prerequisites to successful implementation of alternate assessment processes.

Educational and assessment policies are constantly changing but the major purpose of assessment will continue to be *to inform teaching and learning* and as such will impact on the kinds of tasks and processes we use for assessment, the timing and use of such tasks and processes, the way the students view assessment and the subsequent quality of our teaching.

The most effective teachers are likely to be those who approach assessment as an opportunity for students to show what they know and can do, who link assessment, learning and teaching and regard assessment as a prelude to action.

So changing classroom assessment is the beginning of a revolution- a revolution in classroom practices of all kinds. A tall order, but not an impossible one! Over a period of time, teachers, parents and students can re-form the nature of assessment in school from a culture of judging and categorising to one that fosters learning for all.

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