Reflecting on Mathematics Teaching Situations: A Comparison of Pre-service Mathematics Teachers' and Mathematics Teacher Educators' Views

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Abstract

The low uptake of Higher Level mathematics (highest level examination in the Irish system) and the large failure rate of mathematics in secondary school state examinations each year have highlighted the need for reform of mathematics education in Ireland. A move towards addressing the teaching and learning of mathematics is being implemented under the new initiative 'Project Maths' which sees greater emphasis placed on student understanding of mathematical concepts and applications. Change must also occur in the training of our mathematics teachers since research internationally has shown that poor conceptual understanding often exists among pre-service mathematics teachers. This study uses critical reflection of video-based experiences and live observations to develop pre-service mathematics teachers' awareness of their subject content knowledge, as well as their pedagogical content knowledge. This paper compares the mathematics teacher educators' reflections to that of the pre-service mathematics teachers' reflections. The findings and implications for mathematics teacher education are discussed.

1. Introduction

In recent years, there has been much research into the importance of subject matter knowledge for teachers [1]-[3]. Even [1] concluded that many studies investigating mathematics teachers' subject-matter knowledge advise that often secondary school mathematics teachers do not have a sound understanding of the mathematics they need to teach in school. The research suggests that this lack of knowledge and conceptual understanding applies to pre-service mathematics teachers also. According to [4] pre-service teachers tend to focus on the learning and application of routine procedural skills.

Poor teacher knowledge is a national problem as well as an international problem. The National Council for Curriculum and Assessment (NCCA) [5] reported that the focus of teaching at secondary school in Ireland is on memorising and routine performance. PhD work by the author suggests that first year pre-service mathematics teachers at the University of Limerick (UL) have a fragmented, disjointed view of mathematics and the approaches that they adopt to learning mathematics are mainly of a procedural nature.

Teacher subject matter knowledge cannot be looked at in isolation of teacher pedagogical content knowledge. As [6] explains, no matter what their intentions are, many pre-service teachers do not have adequate mathematical conceptions or know-how to employ effective teaching strategies in the mathematics classroom.

This study was undertaken in order to critically assess, reflect and importantly, develop pre-service secondary school mathematics teachers' awareness of their subject content knowledge as well as their pedagogical content knowledge. This was done using video-based experiences as well as live, in-class observations.

The study is predominantly exploratory in nature since it aims to identify and create awareness among pre-service mathematics teachers' in relation to their subject and pedagogical content knowledge. Although not analysed in this study, it is also hoped that in light of their Knowledge Quartet (KQ) training outlined in the methodology below, pre-service teachers will develop their knowledge in preparation for their video-based teaching experience and indeed after this experience. Shulman [7] clarified subject matter knowledge as knowledge of the content of the discipline such as facts and concepts. He described pedagogical content knowledge as the manner in which a teacher can represent the subject in a way that others can comprehend and an understanding of what makes the learning of the subject easy or difficult.

2. Initial teacher education in Ireland

In Ireland, initial teacher education is in the form of both consecutive and concurrent programmes. For secondary school teachers the consecutive route is a

one year professional diploma in education (PDE) while the concurrent programme runs for four years. The pre-service secondary school mathematics teachers in this study are part of a four year undergraduate concurrent teacher education programme in Physical Education and Mathematics. At UL, where this study was conducted, the pre-service teachers have a number of mathematics content modules each semester over the four years. They study two mathematics pedagogy modules – one in the second year of the course for 6 weeks and one in the fourth year of their course for 13 weeks.

Initial teacher education plays a vital role in the development of pre-service teachers and their mathematical knowledge. According to [8], mathematics teachers' knowledge of how students think and reason is vital to the pedagogical content knowledge of that teacher, informing their instructional practices and decision-making within mathematics tasks. He thus concludes that college and university mathematics teacher preparation programmes should desire to build this knowledge base. Reflecting on teaching situations is also strongly advocated in teacher education programmes and is particularly emphasised for initial teacher education in Ireland by the Teaching Council (the professional regulatory body for teaching in Ireland). The importance of reflecting in mathematics teaching is now outlined.

3. Role of reflection in mathematics teaching

Reflecting on one's teaching is often highlighted as a fundamental aspect of a teacher's training and professional development. Teacher educators need to progress beyond teaching general reflective practices and instead provide pre-service teachers with opportunities to reflect on teaching in a disciplined and structured way [9]. Many researchers [e.g. 10] highlight the importance of teachers learning through analysing teaching. It is important that pre-service teachers know what they should look for and have guidance in terms of what they should notice in a mathematics lesson. Differences often exist between what teachers attach importance to in a lesson and what students attach importance to [11] and the same may be said of teacher educators and pre-service teachers. Video-based experiences using a specific analytic framework offer a clear, structured method to reflect on teaching situations. Researchers such as [12] identify the effective use of case studies and video clubs to motivate pre-service teachers to critically reflect upon professional issues.

4. Video-based experiences

Research carried out on the use of video-based experiences in the training of pre-service teachers as a tool to critically observe and reflect on their own teaching and the teaching of others has presented many positive findings. Video-based experiences provide students with an opportunity to reflect and review their, and others, mathematics teaching, helping them to become aware of their practice as well as assisting them to grow in their pedagogical content knowledge [13]. Video-based experiences are also identified as a rich pedagogical tool that can foster positive attitudes towards learning, and promote learning by increasing students' motivation [14]. Mathematics teacher educators and their pedagogy classes are paramount to the success of this continued move towards a more constructivist approach to teaching and learning. Teacher educators have a responsibility to support this process of change by exposing their pre-service teachers to innovative learning environments [15] such as the use of video. Live observations were also used in this study.

3. Theoretical Framework

The theoretical framework employed in this study for investigating pre-service mathematics teachers' subject content knowledge and their pedagogical content knowledge, was the KQ. The KQ was chosen since it focuses pre-service teachers' attention on important aspects of mathematical content and pedagogical content knowledge during a lesson.

Rowland and his colleagues [3] created this framework for the observation and review of mathematics teaching. It consists of four units: foundation; transformation; connection and contingency. Each unit is subdivided into smaller sub codes of which there are 17 in total. The framework used in this study is a slight adaptation to Rowland et al.'s [3] KQ since one or two of the sub codes are not identical to the original version e.g. "depth of mathematical knowledge" was not a code in the original KQ. Table 1 overleaf summarises the KQ framework used in this study.

Foundation is described by [16] as trainees' knowledge, beliefs and understanding acquired in preparation for their role in the classroom. Transformation concerns knowledge-in-action which is demonstrated both in the planning to teach and in the act of teaching itself.

Table 1. The knowledge quartet (adapted from Rowland and his colleagues [1 and 3])

Foundation	Transformation	Connection	Contingency
-Depth of mathematical knowledge	-How the mathematics is communicated to the learner	- Making connections between mathematics concepts	-Ability to think on one's feet
-Use of terminology -Use of textbooks	-Example Choice (real-life examples, other subject areas etc.)	-Making connections between mathematics procedures	-Response to unexpected
-Reliance on procedures	-Analogy -Demonstration -Representation -Illustrations	-Sequencing of subject matter -Anticipation of complexity (awareness of areas which students find difficult.	-Deviate from lesson plan if advantageous

Connection, the third category, links together choices and decisions for the more discrete parts of mathematical content. It includes making connections between concepts and procedures as well as sequencing of subject matter. The final category, contingency, incorporates the pre-service teachers' ability to respond to students' ideas and think on one's feet.

4. Methodology

The research design, research sample, data collection and data analysis is now outlined.

4.1 Research design

The research was carried out in two main stages. Stage 1 – The author and her colleague and fellow teacher educator separately watched and compared three extracts from the 1999 TIMSS video study (selected from the English speaking countries and agreed on by both teacher educators) and analysed them according to the KQ identifying aspects of each of the four units that impacted on the lesson. Following this, pre-service teachers were provided with KQ training in the form of two workshops. The first workshop included a 20 minute lecture and discussion of the KQ. They then observed and analysed one extract from the TIMSS video study in the same manner as that previously done by the teacher educators. This was done individually firstly, following by paired and whole class discussion on findings. Finally, the pre-service teachers were provided with a copy of the teacher educators' analysis to compare and contrast their own analysis with. The pre-service teachers had one further workshop where they again observed and reflected on a 30 minute extract from another TIMSS video and in pairs/groups they discussed what they had observed and reported on based on the KQ units and sub codes.

Stage 2 - The pre-service mathematics teachers were split into pairs for teaching a 50 minute support tutorial (25 minutes each) to "Access" mature students. As the name suggests, these tutorials support the work done in lectures and in the students' regular tutorials provided by the University. Each pre-service teacher was required to teach a support tutorial and reflect on it once provided with the DVD of the lesson. They were also required to attend two other lessons observing and reflecting on their peers' teaching.

The KQ is the basis for all observation and reflection. Pre-service teachers were provided with a copy of the KQ table and a critical observation and reflection report form devised by the author for this analysis (available from the author). Prior to teaching, each pre-service teacher was provided with a tutorial sheet from the lecturer of the Access Mathematics or Access Statistics course with specific questions to follow. The author also attended all support tutorials and observed and reflected on each pre-service teaching session.

4.2 Research sample

The sample included 29 pre-service secondary school mathematics teachers studying Physical Education and Mathematics, a four year concurrent teacher education programme, at UL. These students are high achievers and the majority of the class have received high mathematics grades upon entering university. Access to the sample was not a problem since their mathematics pedagogy module from January 2010 to May 2010 was taught by the author's colleague. This colleague was also the manager of the Mathematics Learning Centre in UL and assigned tutors for all support tutorials. This provided us with the sample to teach – mature students studying an access certificate course designed to refresh students' skills in areas such as basic mathematics and statistics. Many of these mature students have not studied any form of mathematics in a number of years and are returning to education in the form of this Access course which provides a means of entry to third level undergraduate courses in the future.

4.3 Data collection

Prior to data collection, consent was obtained from all mature students and pre-service teachers for recording the lessons. Permission was also sought from the mature students to allow additional pre-service teachers to sit at the back of the class for live observations. The participants were informed that all data was confidential and that it would be stored securely for the author's and her colleague's use only. Four pre-service teachers taught per week, two at the mathematics support tutorial and two at the statistics support tutorial with exception of one pre-service teacher who taught a full lesson on his own due to odd numbers. All lessons were recorded and DVDs developed of each lesson. Each pre-service teacher was given the DVD of their teaching only. The author also had a copy of the DVD of each lesson.

4.4 Data analysis

This paper reports on all stages of data collection. That is, the teacher educators' analysis of the 29 pre-service teachers' teaching, the pre-service teachers' analysis of their own teaching (28 pre-service teacher reports) and the pre-service teachers' analysis of their peers' teaching (29 pre-service teacher reports). The teacher educators' reflection and analysis is directly compared to that of the pre-service teachers.

The author attended each lesson and at a later date she again reflected on all video-recordings in more detail. Again, the KQ and the codes designed by [3] was the framework for this analysis. On completion of this stage of analysis, the teacher educators determined the main themes or findings that emerged from the data under each of the four units. The findings coming from the data were compared by the two teacher educators for consistency.

The pre-service teachers' own critical observation and reflection reports as well as their reflection of their peers' teaching (live observations) are also analysed and discussed in this paper. The pre-service mathematics teachers watched the DVD of their own teaching and reflected on it by identifying aspects of each of the four units of the KQ that impacted on their lesson. The reports were then submitted to the author. One pre-service teacher failed to submit his report so analysis is based on 28 pre-service teachers.

For observation of their peers' teaching, each pre-service teacher attended two lessons (preferably one mathematics lesson and one statistics lesson) and took notes during the live observations, again using the KQ as a framework. They were given one week to submit their reports to the author. Again, the teacher educators determined the main themes/findings emerging from all pre-service teachers' reports in terms of their awareness, or otherwise, of aspects of the KQ that impacted in any way on the lesson. It should be noted here that the pre-service teachers observed different live lessons.

For each analysis, the students were categorised under the four units as "good", "poor" or "average". "Good" was awarded when the majority of instances demonstrated an awareness of effective or ineffective aspects of the KQ. The opposite was true for the "poor" category. Those who showed approximately an equal number of instances of "good" and "poor" reflections were classed as "average".

5. Findings and discussion – comparison of teacher educators' views and pre-service teachers' views

The findings and discussion in terms of both the teacher educators' critical observation and reflection of the pre-service teachers and in terms of the pre-service teachers' critical observation and reflection on their own teaching and their peers' teaching, are now presented and discussed under the four main categories of the KQ; foundation, transformation, connection and contingency. The views of the teacher educators and the pre-service teachers are compared under each heading.

5.1 Foundation

The depth of mathematical knowledge demonstrated by the pre-service teachers was identified as "poor" by the teacher educators. In general, the pre-service teachers relied on procedural knowledge (19 out of 29), described by [17] as instrumental understanding or knowing the 'how' rather than knowing the 'why'. For example, one pre-service teacher did not relate to students' method for calculating the median as it differed from her method. She was confused stating that

> "5.5 is the median because it is the right middle number. I don't know why the formula isn't working".

Many explanations were also focussed at a procedural level. Mason and Spence [18] stress the failings of rehearsal and practice of techniques. While a mixture of both procedural and conceptual understanding is important, an overreliance on procedural understanding can be damaging. A concluding remark from one teacher was that

> "If you can learn off the formulas you'll be fine".

This may have been the manner in which this pre-service was taught himself in school. Developing subject matter knowledge is essential for these pre-service teachers since improvements in particular kinds of subject matter competence contribute to better analysis of practice thus improve teaching [10].

In addition, 25 out of the 29 pre-service teachers frequently used poor mathematical language or failed

to introduce mathematics terminology in their teaching.

In terms of pre-service teachers' reflection on foundation knowledge, there was a mix of awareness evident. 10 out of the 28 (pre-service teachers' own reflection) and 9 out of the 29 (pre-service teachers' reflection of their peers) pre-service teachers recognised strengths or weaknesses in terms of depth of mathematical knowledge, use of mathematical terminology and reliance on procedures. One pre-service teacher, who was categorised by the author as having poor foundation knowledge, recognises this to be the case when reflecting on his own teaching and reflected that he

> "Doubts own knowledge of the content while explaining (solve $64^{\frac{2}{3}}$) to the class, thus creating confusion for the pupils".

When reflecting on their peers teaching, 8 out of these 10 pre-service teachers identified the impact their peers' knowledge and focus on procedures had on the lesson. One pre-service teacher reflected that his peer relied on procedures and constantly referred back to the 'step-by-step' breakdown of a question. While this pre-service teacher felt such reliance on procedure was a downfall, another teacher when reflecting on his own teaching was of the opinion that it was to the students' benefit.

> "I relied heavily on procedures which I felt suited the students".

Overall, 17 out of the 57 pre-service teacher reflections were categorised as "poor" (9 from pre-service teacher's own reflections and 8 from reflecting on their peers' teaching) and findings suggest that this was due to a failure to recognise where teachers relied solely on procedures or displayed poor content knowledge. For example, one pre-service teacher described her peers' knowledge as "great" because of their clear explanations. This is in contrast to the teacher educators who observed that in this lesson, the teachers had struggled to understand the median and provided an incorrect method and solution to the students (prior to intervention from teacher educator present). This, along with similar statements from both pre-service teachers' reflection of their own teaching and their peers' teaching, suggests that their beliefs about knowledge do not always match their content knowledge. This is something that is recognised in the literature (e.g. [19])

5.2 Transformation

Communicating the mathematics to the learner is one of the sub codes under this category. 8 out of the 29 pre-service teachers were categorised by the teacher educators as "good" transformers or communicators of knowledge, 14 as "poor" and 7 as "average". The pre-service teachers were categorised according to the way in which the teacher transformed his or her own meanings and descriptions of the content. An example of where knowledge was transformed effectively through representation and indeed by using a real-life example was where a pre-service teacher used a You Tube video of a car overtaking another car to introduce the concepts of velocity and acceleration. While the author is not suggesting that real world contexts are the only effective way to transform the mathematics to the learner, she is in agreement with researchers such as [20] that some real world connections develop students' understanding of mathematical concepts. 17 out of the 29 pre-service teachers used no real-life context in their lessons.

There was some varied use of demonstrations and analogies but at times these were incorporated in the lesson to little effect e.g. one teacher used the interactive software package 'GeoGebra' to introduce the meaning of differentiation but she struggled to explain what was happening on the diagram. The importance of foundation knowledge is again to the fore here.

Of the pre-service teachers' reflection reports, a greater number (20 out of the 57) were categorised as demonstrating good critical reflection skills in terms of transformation of knowledge (11 from pre-service teachers' own reflections and 9 from reflecting on their peers). These pre-service teachers were very much aware of the way in which the knowledge was communicated, be it in an effective way or otherwise. One pre-service teacher criticises his lack of use of real life examples or teaching in context.

> "Practically no real life examples used throughout the lesson; resulted in pupils not having any context for the content they were being taught".

Another pre-service teacher, when reflecting on his peers' teaching, highlighted the good attempt on the teacher's part to relate statistics to real life by using a Premier League football table before criticising the teacher for not putting the solution back into a real-life context that the students could relate to. 20 out of the 57 reflections were also categorised as "average". The main reason for this was that while the pre-service teacher could identify good or poor aspects of transformation, he/she showed no evidence of any understanding or justification as to why this was the case.

A lower number of pre-service teachers were categorised as "poor" transformers of knowledge when compared to the teacher educators' reflections. These teachers displayed a lack of knowledge and a poor understanding of effective teaching strategies. One pre-service teacher claimed that by asking the students 'what is interquartile range?' her peer was demonstrating effective transformation of knowledge by allowing the students to think for themselves. According to [21], while teacher knowledge of

mathematics alone does not lead to effective teaching performance, it is most certainly a factor in teaching strategies and student learning.

5.3 Connection

The main finding to emerge from this category in terms of making connections between concepts or procedures was that many of the pre-service teachers lacked the knowledge to do just that. In a number of lessons, in particular the statistics lessons, it was observed that no link was made between the answer obtained and the actual concept involved. For example, in one lesson the pre-service teacher begins the lesson by listing some numbers and calculating the mean of those numbers. He then asks "what if you were asked to find the standard deviation of that?" without making any link between measure of centrality and measure of variability. In fact he then answers his own question and tells students "well we fill it into our table don't we?". After progressing quickly through the method for calculating standard deviation he moves onto the procedure for calculating the median, again omitting to make any connection between concepts or procedures. Research has reported on a lack of understanding of the mathematical knowledge necessary to teach well [22].

Sequencing of subject matter is another sub code within this category but as students were provided with tutorial sheets prior to the lesson the code was less relevant to this study. It was interesting to note however, that only four of the 29 pre-service teachers re-ordered the sequence of exercises or topics according to what they felt would most benefit the students' learning. There were eight incidents observed where the pre-service teachers anticipated difficulties their students may have with a particular concept.

This category was the most poorly reflected on by the pre-service teachers. 16 out of the 28 demonstrated very poor critical analysis and reflection of their own teaching and 14 out of 19 were also categorised as "poor" when reflecting on their peers' teaching. It seemed that the pre-service teachers were unaware of what 'connection' means. Another possible reason for the poor reflection was the focus of some pre-service teachers on procedural knowledge. According to one student he had linked and made connections between the mathematical procedures because he had

> "Reiterated to pupils the importance of showing all workings in case you make a mistake".

When reflecting on their peers' teaching, many of the pre-service teachers failed to mention any connection between mathematical concepts and procedures. 11 of the 14 pre-service teachers categorised as "poor" referred to sequencing being good in a lesson when it followed a particular order – teacher explanation, teacher runs through an example followed by student practice. The importance of teachers' beliefs comes into play again here.

Only 5 out of the 28 pre-service teachers' self-reflections and 2 out of the 29 reflections on their peers' teaching demonstrated good critical awareness in terms of connection.

5.4 Contingency

The final category in the KQ is contingency which is determined by the pre-service teachers' ability to think on his or her feet, respond to the students and deviate from the lesson where he or she feels it would be beneficial for one or all learners. The main findings emerging from this category were that most pre-service teachers (21 out of the 29) appeared to lack the content knowledge necessary to interpret students' questions and misconceptions and to confidently deviate from the lesson plan to explain such misconceptions. For example, in a lesson on differentiation one particular student wanted to know how you know to use the quotient rule or to just find a common denominator. She appeared to associate all fractions with a common denominator regardless of what the question was asking. The pre-service teacher did not identify the student's misconception and struggled to answer the question. The importance of subject content knowledge has been reported many times throughout this paper and is very evident here.

There was however, some evidence of good responses and ability to deviate from lesson plan where appropriate and beneficial for students (6 out of the 29). One example of where this was done effectively and to the benefit of the students was when a pre-service teacher deviated from her lesson to recap on the difference between qualitative and quantitative data since it was evident that they were struggling to understand these terms. She uses examples that the students can relate to which clarifies their misconceptions.

There were once again varied pre-service teachers' critical reflections for this category with 11 out of 28 (reflecting on their own teaching) compared to 7 out of 29 pre-service teachers (reflection of their peers' teaching) demonstrating good critical awareness of where contingency was in action or where it could have been improved upon. 8 of the 11 and 4 out of the 7 pre-service teachers admitted that they lacked the content knowledge to deviate from their lesson plan or respond effectively to students' questions.

The teacher educators noted that foundation knowledge seemed to impact on the pre-service teachers' contingency. Of the 17 pre-service teachers overall who reflected poorly on their contingency and their peers' contingency, 10 were not aware that

insufficient foundation or content knowledge hindered their ability to respond to the unexpected.

6. Conclusions

The above findings offer an insight into pre-service teachers' subject and pedagogical content knowledge from the teacher educators' perspective and from the pre-service teachers' perspective. Some findings suggest that pre-service mathematics teachers may not have sufficient subject matter knowledge to alter their teaching strategies and ultimately teach for understanding. However, pre-service teachers' reflections of their peers were slightly more encouraging than their reflection of their own teaching. Many of the peer reflections took place further into the semester and the pre-service teachers' ability to reflect may have improved.

There is an onus on mathematics teacher educators to develop pre-service teachers' ability to move away from traditional approaches to teaching and create an awareness of the benefits of doing so. Video-based experiences are one way of doing that and this is one of the main reasons as to why the data was firstly analysed from the teacher educators' perspective. From their own reflections and the findings reported above, it is clear that there is mixed awareness among the pre-service teachers about the need for them to develop these essential tools for effective mathematics teaching. This offers a major challenge for teacher educators to plan and develop their courses to include innovative learning environments necessary for change in the teaching process to occur.

Video-based experiences and in-class, live observations also provide trainee teachers with the opportunity to develop critical reflection skills in terms of both their own teaching and their peers' teaching which is why this study also included analysis from the pre-service teachers' perspective. Evidence from the paper highlights the further need to enhance pre-service teachers' reflective skills and develop their awareness of how to so do effectively.

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