Can Economic Knowledge Be Created in Classrooms? Some Insights from Epistemologically-Oriented Interaction Analysis: Mathematical Registers and Metacognitive Mediation

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Abstract

The semiotic and metacognitive dimensions of the Mathematics-Economics alliance are studied to understand the influence of the nature of Economic Education on the nature of Economic Knowledge. Along with the representational character of Mathematics in Economics, one of the features of this long-lasting relationship not explored enough so far, is that the way higher order thinking skills are applied in Mathematics for solving problems, could be used in Economics learning to develop the same sort of skills with direct reference to real-life situations. Being inherently relational, mathematical registers and knowledge structures enable the transferal of problem solving skilfulness from Mathematics to Economics through epistemologically-oriented classroom interaction. Semiotic conceptual thinking underpins this knowledge creation as a collective-meaning creation process. A model called the 'Learning Hexagram' emerges.

1. Introduction

The semiotic and metacognitive dimensions of the Mathematics-Economics alliance are studied with the aim of understanding the influence of the nature of Economic Education on the nature of Economic Knowledge. Along with the representational character of Mathematics in Economics, one of the features of this long-lasting relationship not explored enough in the Economic Education literature, is that higher order thinking skills applied in Mathematics for solving problems ([3], [21], [36]), could be used in teaching-learning design to develop the same sort of skills with direct reference to reality.

In fact, the consolidation of these skills becomes decisive in higher education, consequence of increasingly competitive job markets and much higher governmental funding demands [35]. The present research highlights a different approach to the traditional Economics-Mathematics relationship, which can have an upfront impact on the way Economics is predominantly taught in both College and University. It is felt that being able to make soundly informed decisions is vital to students aiming for a career in Economics and linked disciplines such as Accounting, Business and Finance, either at policy-making or management practice level.

1.1. Rationale

It makes sense to assert, the way students are taught to think as economists will eventually reflect on the way they apply economic knowledge to solve real-life economic problems. The closer this knowledge is to reality, the more effective solutions to relevant problems are going to be. This is such simply because being social, Economics deals with human activities that involve people, and being a Science, this ought to be seeking for the understanding of phenomena inherent to individuals that interact to produce, consume and distribute goods and/or services.

Therefore, cognition in Economics learning should be drawing from these phenomena. This is so because its epistemic content is to be found in quotidian events. Indeed, the Heideggerian idea of "being-in-the-world" encapsulates the ontology here considered paramount to learning processes: Human beings spontaneously use their rationality in everyday life [22] and this is what they bring to the classroom.

Hence, economic knowledge needs to emerge from these everyday situations, which are ultimately man-made and consequently human-biased. The semiotic power of Mathematics ([7], [14]) is what may allow for diverse representations of the contingent reality in order to understand it, call the 'dialect' School-learnt, Neoclassical, Bayesian, etc.

Most human activities result in a final product: Voting for a political party, doing the garden, building a grass-roots refinery, going into war, trading stocks, having a baby, managing public deficit, undergoing radiotherapy, etc. The fact is, resources are scarce and decisions are to be made with the purpose of achieving objectives given there are alternatives, may the goals be materialistic or altruistic.

The approach here undertaken assumes the existence of complex social contexts: The aim is to design a mechanism that reflects them when these are economically analysed. Its main contribution lies on the development of a teaching-learning model intended to capture economic concepts through the social semiotics of Mathematics by using epistemologically-oriented interaction analysis.

1.2. The prospective teaching-learning model

Undoubtedly, Economic Education plays a major role in building up the skills an individual should develop to make professional decisions on scarce resources allocation with alternative uses and multiple ends. In the following paper it is Economic Education what drives not only the ability of students to solve economic-inherent problems beyond classroom, but also the underlying epistemology.

From the angle here adopted, economic phenomena can be dissected through classroom interaction analysis and then reconstructed as theory (or theories) with collective meaning. Precious insights are gained from modern Mathematics Education ([33]), which along with the Mathematics discipline has undergone a major philosophical transformation in the last half-a-century ([8], [9], [10]).

The thesis is that problem-solving in Mathematics is transferrable to Economics through metacognitive skills, the reason being that when students take on Economic-related subjects for the first time, they have already been trained up-to-some-extend to 'think economically' because of compulsory school Mathematics; in fact, this learning process constitutes a knowledge structure. The challenge lies on making the epistemic transition. It is here sustained the best way to do it is through word and modelling problems by prompting metacognitive questions that elicit interaction. Then knowledge is to emerge in the social semiotics of collective meaning creation.

There are two significant findings. First, because of its relational character Mathematics is established as a representational language for Economics in reality sense-making. Second, Mathematics and Economics share a common pedagogic feature: Long term abilities involve metacognitive training. In this respect, knowledge is here found to be a living reconstructing body that requires higher order thinking; besides, the more complex the problem is the more useful social interaction can be. These outcomes are crystallised in a theoretical model called the Learning Hexagram.

In short, Mathematics is established as a language that can be used to develop metacognitive skills in economic-related disciplines to create reality-referenced collective meaning and therefore knowledge in a community of practice. Moreover, word and modelling problems are pivotal for metacognitive questioning mediation, which is central to the teaching-learning system embedded in the Learning Hexagram.

The following paper exploits the framework above sketched as a result of the application of epistemologically-oriented interaction analysis ([29], [31], [33]) to Economic Education. This is not known to have been done previously in the relevant literatures; hence the cross-disciplinary intent of pedagogic innovation in Economics can be seen as a broader contribution.

2. Knowledge and Education: Mathematics and Economics from the Interactionism perspective

2.1. Background

The departure point of the approach here undertaken emerges from Mathematics Philosophy, and is the notion that Mathematics is "the product of organized human activity, over the course of time" ([10], p.269); a direct consequence of this premise is that "mathematical knowledge cannot be constructed independently from a social development context" ([33] , p.12).

According to Steinbring [33], academic mathematicians and Mathematics educators carry out their activities in similar social contexts where knowledge sharing through communication is crucial; therefore, these two communities follow similar epistemological paths. However, the outcomes of their activities may differ considerably in the purpose of learning, and consequently in the degree of formalization, basically because of the cognitive development of individuals.

It is inferred from this assertion that classroom interaction and therefore mathematical communication are mechanisms culturally shared by academic mathematicians and Mathematics educators; then, it can be stated that since these communities follow similar communication dynamics, the depth of their discernments may get to differ, but not the manner mathematical knowledge would be created.

Steinbring ([33]) goes further and proposes that logical and ontological forces interact in the construction of mathematical knowledge. This then implies two complementary actions. The first one is connecting existing relations conventionally accepted with logical interpretation by stating them, which leads to equality and correctness, but without insights, and therefore, without comprehension. The second one, called 'ontological understanding' ([33] , p.69), is understanding by finding and interpreting new relations, which is going to be the outcome of individual introspection, very likely triggered through social (classroom) interaction. Both actions may happen at any cognitive developmental stage, from pre-school to doctoral level.

Both actions are interdependent, because in the absence of logical structure mathematical ontology is meaningless, and understanding on its own is not a valuable device for either research or life-long learning. It is hence interpreted that logical and ontological understandings link in the collective creation of mathematical meaning between mathematician and educator communities, at different complexity levels.

To illustrate the interaction between logical and ontological forces, Steinbring ([33], [34]) develops what he calls "the Epistemological Triangle", which is an analytical instrument that, according to this author, relates objects/reference context, sign/symbol and concept. An adaptation for the analysis of economic phenomena is[1]:

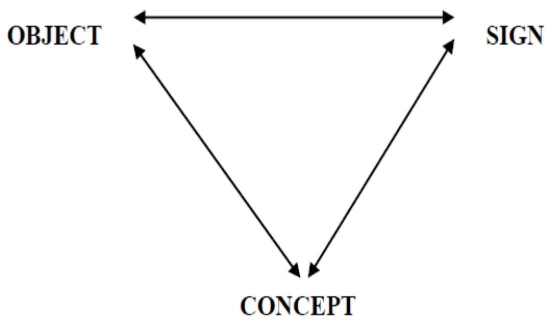

Fig. 1 the Epistemological Triangle (adapted from [33], p. 22)

Observe this triangle reflects the principle according to which mathematical knowledge is essentially an open process, not a mere product ([12], [28], [31], [33], [34]). Accordingly, from the Interactionism's perspective, in classroom communication there is no room for ready-made knowledge transmission, since this is conceived as an active mediation process in which individuals contextualise concepts in terms of signs by logical and ontological understandings, giving place to mathematical relations perceived as new at individual and eventually at collective level, depending on their degree of complexity.

In the triangle, object denotes any 'external' image that may be used to exemplify the mathematical problem (usually a tangible or graphic idea), sign denotes the way the problem is translated into (letters and/or numbers), and concept denotes any new mathematical relations found through semantic constructions.

2.2. The semiotic power of Mathematics

A key difference between Economics and Mathematics can be derived from above: Mathematical objects are defined relationally ([11]), in contrast, economic objects are defined in terms of themselves. For example, a random number differs from a common stock in the fact that one cannot observe the defining property of random number from outside as an attribute, whereas this is visible in a common stock certificate. However, a common stock price could be studied as a random process. This distinction involves deep implications for Economic Analysis, because it underlines the representational character of mathematical language as a semiotic instrument.

The semiotic dimension of Mathematics has been recently re-explored, amongst others, in [6], [7], [14], [24], [25] and [26]. In the present paper, Semiotic Epistemology is found to be fundamental for Interaction Analysis. Particularly relevant is Peircean Epistemology, as exposed in [24]: "…To know means to relate a particular experience to a concept…A particular or individual, by being represented and by thereby being transformed into a sign, becomes a general…" (pp. 17-18). For Economic Education, this view opens a whole new avenue in knowledge creation:

"…As all general phenomena are fundamentally semiotic entities, while singular phenomena are not intrinsically signs, we could also say that Epistemology is concerned with the relation between the singular and the general. In this way generalization appears as a fundamental problem of Epistemology and Education…" ([24], p.17)

How can this new avenue be travelled through in Economics? A way could be by assimilating in figure 1 the concept of semiotic systems, as defined in [7]:

"...The term semiotic system is here used to comprise three necessary components. First, there is a set of signs, each of which might possibly be uttered, spoken, written, drawn, or encoded electronically. Second, there is a set of rules of sign production, for producing or uttering both atomic (single) and molecular (compound) signs...In Mathematics this includes rules that legitimate certain text transformations, e.g. 'cancelling', the common division of numerator and denominator in fractions. Third, there

[1] Modifications are made to support Peircean Semiotic Epistemology (see next sub-section)

is a set of relationships between the signs and their meanings embodied in an underlying meaning structure" ([7], pp.69-70)

Ernest [7]'s analysis points out that in school Mathematics semiotic systems are transformed, being the final product part of the contents of school Mathematics: Number and Algebra constitute a semiotic system, as well as it does Geometry and Measures or Statistics. Analogously, this can be extended to Dynamic Programming, Stochastic Analysis and Graph Theory, some few examples of semiotic systems transformed at Higher Education and/or academic research level.

Again, mathematician and educator communities partake in these transformation processes. In fact, these are social semiotic processes, namely, "…meaningful practices, arguing that meaning emerge from an on-going and implicit negotiation between different parties involved in a common context" ([22], p.31). Academic researchers and educators in Economics share analogous communication cultures. Thus by relating economic phenomena to concepts through mathematical representations, Economic-related knowledge becomes a collective meaning-creation process.

Now, in the epistemological triangle, the mathematical sign conventionally conveys "…a semiotic function: the role of the mathematical sign is 'something which stands for something else'" ([33], p.21). Then this is interpreted as part of a transformed semiotic system: From this paper's viewpoint, a transformed semiotic system is a 'dialect' through which economic phenomena can be represented, so that mathematical symbols are used to relate economic meaning to economic phenomenon. Illustrating in figure 2, "R" denotes the dividend yield related to the stock certificate shown in the picture. Concept "Dividend Yield" and "R" interrelate (↔) to constitute a transformed semiotic system that conveys an economic meaning: Dividend Yield.

Thereby the existence of mathematical symbols and the rules for producing them is linked to the existence of economic objects because there is a meaning to be understood both logically and ontologically. This underpins the relational character of Mathematics in Economics: In the creation of collective meaning, the use of mathematical symbols will depend on the ontological understanding of economic phenomena by finding and interpreting (new) relations among entities.

Thus, in figure 2, another transformed semiotic system could be Corporate Governance ↔ Linear Compensation Incentives or Portfolio Choice ↔ Mean Variance/NPV Analysis, etc. That is, the same finance entity (in this case, the stock certificate 'as such') can generate different semiotic representations. From a cognitive viewpoint, this notion takes epistemologically-oriented interaction analysis further.

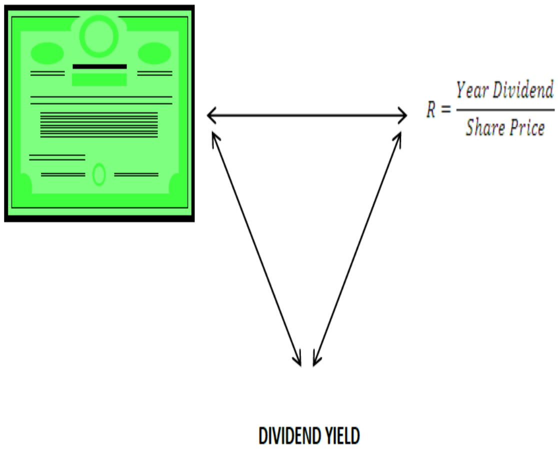

Figure 2

Economic entities can be captured in [7]'s semiotic systems by using Duval's mathematical register definition, according to which "not all semiotic systems are registers, only the ones that permit transformation of representations" ([6], p.111), first through conversions and then through treatments. Conversions in Economics are here conceived as oral, written and/or graphic accounts of economic events translated into mathematical symbols, and treatments are considered any transformation thereafter, e.g., dynamic programs, cointegration models, indexes, etc.

In figure 3 below the semiotic system Options on S&P Index ↔ M_t can be regarded as a treatment of some instance of the stock certificate depicted in figure 2, or as a conversion on its own. Consequently, $p(W_t, t)$ constitutes a mathematical register that represents this transformed semiotic system through stochastic calculus[2]. Another mathematical register (susceptible to further treatment, as the ratio $\frac{Year\ Dividend}{Share\ Price}$ above) could be the more 'verbal' conversion: Exercise Call/Put Option ∴ Daily S&P Index ⋚ Monthly Average S&P Index. A key insight is: The latter algebraic-type representation is epistemologically as valid as the previous one; its register is just less sophisticated.

[2] Where M_t denotes a Martingale and W_t denotes a Wiener Process. This graph [30], describes the Real S&P Stock Price Index-Base 1871=100.

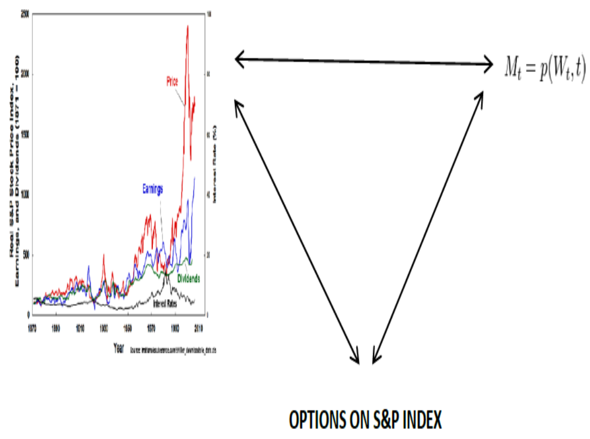

Figure 3

If instead of the stock certificate in figure 2, the object were a grass-roots refinery, school or hospital, other economic concepts might be invoked. The last two entities might be conceived in terms of public policy expenditures or in relation to resource allocation alternatives between public and private services; a common concept to the three entities would be capital investment projects. Therefore, different spheres of phenomenon understanding could be stirred. Observe in fig. 4 below, as part of the example developed so far, Portfolio Choice ↔ Real Options is also a feasible transformed semiotic system of highly sophisticated register, but with different object.

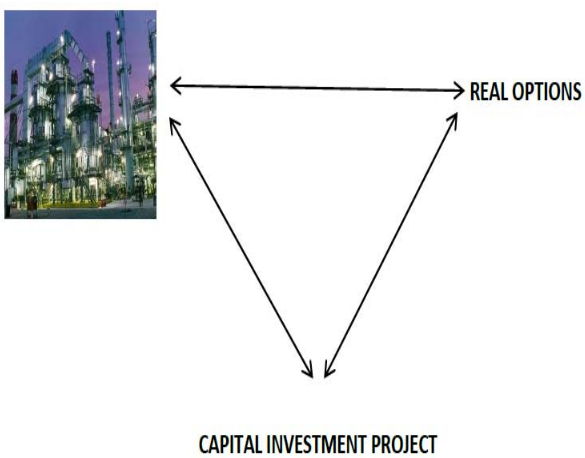

Figure 4

The didactic point is, if for example someone asserts that “Public expenditure cuts reduce fiscal deficit”, this person or another one may also state that ↓G→↓D and equivalently another one can state that $\frac{\partial D}{\partial G} > 0$. The difference is the register level, but the concept is mathematical in essence, even as a ‘pure verbal’ statement. This last assertion will be deepened when knowledge structures are discussed next section.

In pedagogical terms, from the above analysis it is established that diverse individuals in the same classroom could use different mathematical registers to represent the same economic entity and still give place to new ontological representations and therefore create economic knowledge. In a community of practice such as the Economics classroom, communication will then become crucial in this collective-meaning creation process, and so it will do peer interaction and instructional mediation. In the next section, metacognitive questioning will be found to motorise this process, being the implementation of word and model problems central to develop it.

3. Metacognitive mediation in epistemologically-oriented interaction analysis: The case of word and modelling problems in Economics

3.1 Metacognition and Epistemology in Economic Education

An aspect of the relationship between Mathematics and Economics that can now be claimed to be semiotic-inherent is the existence of mathematical ideas such as quantity, measure, change, shape and uncertainty, which makes these two disciplines ‘affine’: It could be underlined that most of economic concepts are prone to be mathematized because the economic phenomenon lends itself to mathematical representation. On account of logical and ontological dynamics, it has been established that mathematical registers can generate economic knowledge. It is now proposed that metacognitive mediation could conduct this process in collective meaning creation.

The representation of economic concepts using mathematical language has been so far the focus of this enquiry; in general, codified knowledge facilitates communication not only within but also between scientific communities [23]. However, no knowledge creation process is complete until the human mind comes into play. The intent here in this sense is quite modest, since the interest is on how cognition occurs when learning Economics and why mathematical registers might be used. Some notions on metacognition are then fundamental.

For the purposes of this paper, a basic distinction is made between metacognitive knowledge and metacognitive skills. According to Veenman [36], in Mathematics learning metacognitive knowledge is declarative knowledge about self-regulation in a learning situation, whereas metacognitive skills are the procedural knowledge required for self–regulation in the same learning situation-that is the reason why metacognitive are by definition higher-order thinking skills.

In other words, metacognitive knowledge is what it is known on how to undertake a learning activity and metacognitive skills is the way in which this is put into practice. Metacognition encompasses these two definitions. Task analysis, planning, monitoring, checking and reflection are expressions of metacognitive skills [36]. It is acknowledged that these abilities need to be triggered in pedagogic activity.

Metacognitive mediation is consequently defined as the development of metacognitive skills through classroom communication, which means that instructor-student interaction and/or student-student interaction (with or without instructor's moderation) give place to what Steinbring [33] calls "communicative funnelling through classroom episodes"; such episodes per se involve social interaction.

There are two sides to be visited in the metacognitive mediation thesis, though. The first one is based on the notion of conceptual thinking and the second one on the concept of evaluativist epistemology, both of them to be framed in the area of cognitive development. In this sense, [13], [18] and [20] emphasize the importance of epistemological understanding in cognitive development processes given that "… educational experiences enhance the progression toward advanced epistemological outcomes…Such outcomes are often viewed as congruent with higher order thinking skills…" ([13], p.46)

On one hand, conceptual thinking in Mathematics, defined after [36] as Mathematics problem-solving geared by metacognitive skilfulness, is in this inquiry mirrored in as conceptual thinking in Economics, on the basis that these two disciplinary fields are subsumed into knowledge structures:

"…'knowledge structure'-an internalised framework of all the related perspectives, concepts, ideas and methods of enquiry making up the knowledge domain and giving it meaning… [represents] a central tenet of cognitive science. The tenet is that the organisation of knowledge is at least as important as the quantity of knowledge accrued in helping the individual to determine when and how a set of declarative facts apply to a particular situation ([5])…" ([16], p.99)

Metacognition in learning motorises problem-solving because conceptual thinking implies relational understanding. This means that reality-referred knowledge can be organised by transferring problem-solving skilfulness from Mathematics to Economics, for conceptual thinking enacts structures that are common to both disciplinary fields, very likely due to their semiotic affinity. In educational terms:

"…learners may ascend to the next stage of intellectual development-the relational level. At this level a learner gives consideration to the relational structure of knowledge. As a result the learner develops an ability to recognise the underlying relationships in the knowledge structure…" ([16], p.107)

Another key insight now argued is that relational understanding involves both logical and ontological understandings, since epistemological understanding can entail different register levels in economic knowledge creation. In this sense, Brier in [2] rescues the symbolic and pre-symbolic character of mathematic Semiotics, in that a 'verbal' mathematical explanation can be as effective as a symbolic one because the underlying logic is pre-symbolic. Hence knowledge structures are by definition pre-symbolic, but still instrumental to any collective meaning creation process.

On the other hand, evaluativist epistemology, as defined in Kuhn, Cheney and Weinstock [18], is subsequently regarded to be the cognitive milestone that should be sought after in metacognitive mediation, since "…at the heart of the evaluativist epistemological position is the view that reasoned argument is worthwhile and the most productive path to knowledge…" (p.325)

However, [18] goes beyond this commonplace and "…propose the coordination of the subjective and objective dimensions of knowing as the essence of what develops in the attainment of mature epistemological understanding…" (p.309). Subjectivity-objectivity coordination may thus pave the way to generalization in the Peircean sense earlier exposed.

Within a broader theoretical approach, these authors sustain that multiplist judgements of truth are subjectively legitimate basically because (in a democratic society) any individual is entitled to a personal opinion, principle susceptible to be translated from daily life into different domains of knowing. They identify this as a level of epistemological understanding:

"In the transition from multiplist to evaluativist [epistemology]…the developmental task is one of recognising and reintegrating the objective dimension of knowing…It may be easiest to recognise the possibility of objective criteria (in the face of multiplicity of views) in the domain of truth judgements: Scientists are recognised as having divergent views, but evidence suggests one scientist's model to be more accurate ("closer to the truth") than another's, with the further possibility of such distinctions being more readily accepted when claims are about the physical world than when they are about the social world…" ([18], p.314)

The last sentence of this assertion is particularly relevant to Economics. Economic concepts tend to be "troublesome" predominantly because of the common-sense notions that learners develop to make sense of their own experience, which is narrow by definition since, for instance, individuals are more likely to be consumers than producers or borrowers than lenders [4]. But this is the real life experience that students bring to the classroom and this is where from economic knowledge should emerge. The thesis here sustained follows [23]'s statement, in that communication will bring up an agreement on what constitutes reality.

Conceptual thinking as a knowledge structure is therefore the mechanism aimed to be triggered in classroom interaction when it comes to reality-referred economic knowledge. Consequently, the transition from multiplist to evaluativist epistemology for epistemological understanding progression in Economic Education is viewed as one correlated to cognition development: From this paper's angle, epistemological maturity can be achieved through metacognitive mediation to support this development process.

3.2. Word/modelling problems and conceptual thinking in Economic-related Education

Next step in this enquiry is how to implement metacognitive mediation in classroom interaction for higher education Economics and related subjects. Resourcing to semiotic conceptual thinking by means of knowledge structures takes this research to the realm of school Mathematics studies, where evidence on the effectiveness of the proposed techniques of word and modelling problems is substantial.

Word and modelling problems are proposed because these are considered by, amongst others, [1], [15], [17], [21] and [27] highly concept-rich activities for school Mathematics instruction, essentially because these compel systematic conceptual thinking. Analogously to Mathematics but circumscribed to the economic-related disciplines contexts, word and modelling problems in Economics are defined as textual problems (usually in writing) replicating real-life pertinent situations.

Ben-Hur [1] has analysed the importance of word problems in the development of conceptual thinking. Yet modelling problems have been considered more challenging than word problems in terms of ability level [21]. According to Mevarech, Tabuk and Sinai: "…Modelling involves translation of 'authentic' problems into mathematical expressions, mathematizing real life situations, or finding patterns and generalisations…" ([21], p.74)

The difference between word and modelling problems may lie though in terms of task difficulty in text comprehension, which in some cases may imply the construction of situational models to approach more difficult word problems, because of the need of integrating existing world knowledge with information derived from the word problem text [15].

Additionally, note metacognitive skills in the planning of Mathematics Literacy for compulsory education are expected to be successfully embedded during adolescence ([32], [36]). Indeed, Sierpinska [32], recalling Vygostsky's Zone of Proximal Development (ZPD), argues that in order to fully develop metacognitive skills, mathematical instruction should step ahead of what might be recognised to be the 'natural' intellectual developmental age during adolescence. It is for that reason inferred that conceptual thinking as a knowledge structure will be hindered if this embedment fails to happen at some point in adulthood.

In fact, some studies have found that:

"Research involving older children, adolescents and adults suggests that meta-knowing competencies [that is, awareness, understanding and management of one's cognition]-in contrast to most of the competencies that developmental psychologists study-remain incompletely developed…" ([20], p. 21)

Therefore, even after completing compulsory Mathematics education, learners may lack some key problem-solving skills. This finding is corroborated in [19] and empirically supported by [18]. There might also be important epistemological understanding limitations due to intellectual functioning [18].

Mevarech et al [21] underlines that metacognitive skills can be successfully trained through school instructional intervention. Nevertheless, if these are not fully developed by adulthood, metacognitive skills might need to be reinforced in higher education. Then for Economic Education, metacognitive mediation can be applied to strengthen conceptual thinking and so catalyse epistemological maturity. Aiding [32]'s ZPD angle, Ivanitskaya et al [16], refer to post-graduates as follows:

"…It may be possible, however, to minimize the distance between students' actual epistemological levels and their potential for development by placing problem-solving activities under the guidance of instructors or encouraging collaborations with more capable peers" ([16], p.104)

Since word and modelling problems induce systematic conceptual thinking because both of them enhance problem-solving through metacognitive skilfulness, if there was a teaching-learning method to be proposed in higher education Economics, it is here sustained that this should be based on the use of these two pedagogical tools. Next, a model named the

"Learning Hexagram", illustrating how Economic knowledge can be created via the development of epistemological understanding.

3.3. The Learning Hexagram

The intuition behind the Learning Hexagram is that once a new concept is introduced, students in word/modelling problem-based classrooms will engage in conceptual thinking more effectively than in pure lecture/discussion-based classrooms, because they would need to 'think and talk more' to solve these problems, in both student-student and instructor-student interaction settings. Thinking-and-talking means communicative funnelling that materialises through metacognitive questioning.

The cornerstone of the Learning Hexagram model is the metacognitive questioning proposed in [21], as follows:

"…meta-cognitive questioning includes four kinds of…questions: Comprehension questions (e.g., What is the problem all about?), Connection questions (e.g., How is the problem at hand similar to, or different from problems you have solved in the past? Please explain why), Strategic questions (e.g., What kind of strategies are appropriate for solving the problem, and why?), and finally Reflection questions (e.g., Does the numeric solution make sense? Can the problem be solved in a different way?)…" ([21], pp.75-76)

Hence comprehension, connection, strategic and reflection questions have to be elicited in the formulation of any word and/or modelling problem for this to qualify as a metacognitive mediation tool. Then key to the role of the instructor (lecturer) is to conduct classroom interaction by prompting metacognitive questioning to make learners 'think and talk more' with the aim of creating collective meaning. In effect, this is to be made explicitly part of the formative assessment.

The leading hypothesis is: If students engage more effectively in conceptual thinking by answering metacognitive questions, then epistemological understanding would eventually mature from multiplism to evaluativism in economic knowledge creation. The process involved is complex, so a pedagogical approach to its application is taken, which is graphically characterised in figure 5.

Being the Learning Hexagram based on Steinbring [33]'s epistemological triangle, and paraphrasing that author from this research's perspective, it is emphasised that economic knowledge cannot be constructed independently from a social development context. In this sense, the Learning Hexagram can be thought of as a learning algorithm for each and every participant in classroom interaction (including the lecturer, as the most advanced learner). In the next section, it will be pointed out how degree of cooperation among participants may vary depending on task difficulty.

The stages followed in a broadly generic session applying the Learning Hexagram can be read below as, A: Starter, B and C: Main session and D: Plenary.

A. Recommended readings made before the session. Instructor introduces a new concept by circulating a pertinent word/modelling problem in writing.

B. The new concept relates to the economic object in a specific manner (see examples in figures 2, 3 and 4 above), which is then addressed. Hence, the instructor needs first to tackle the reasoning as to 'why' the object relates to the new concept, and thus the 'how' will follow, bringing up key vocabulary relevant to the concept. B ought to be concise, ceasing after instrumental terminology is mapped, which might or not include mathematical registers (!)

C. Students solve the problem in writing; they may discuss it in groups of up to three students. Alongside, relevant comprehension, connection, strategic and/or reflection questions are displayed in writing for participants to self or peer-regulate epistemological understanding and so knowledge generation.

D. Written work is produced to be self and/or peer-assessed before session ends.

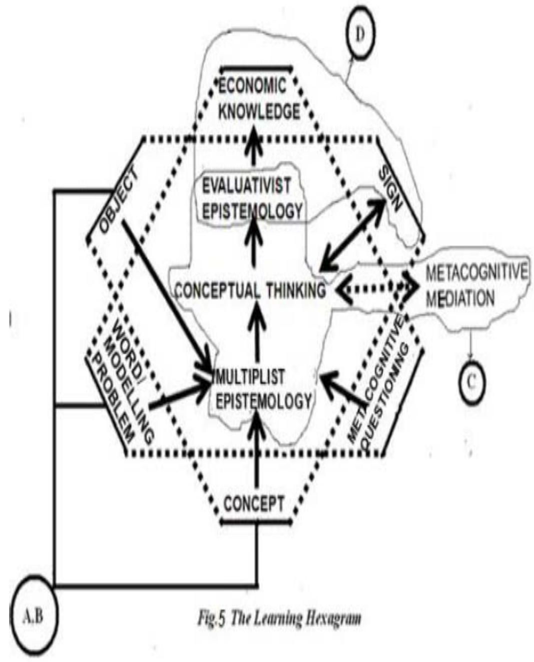

Fig.5 The Learning Hexagram

4. Model Analysis

Needless is to say, there are some fissures to be filled out in the Learning Hexagram model as it is. These are thought of being mostly application-wise and for that reason prone to be revisited under different scenarios. The ones considered to be more challenging content-wise, though, are semiotic-related, namely, the ones in double implication in the right-hand side in figure 5.

There are two important openings from that side of the model that bring up question marks. First, could in-depth economic analysis be done without the use of any mathematical register or by means of less sophisticated ones? Second, are mathematical registers in Economic Analysis more likely to be used in mediator-learner or learner-learner interaction, or none at all? Both queries are interconnected. The argument developed is grounded on a Social Semiotics approach to Economics and its education.

Consistently with Sierpienska [31] interactive communication regulates communal practices in the classroom. Furthermore, challenging questions develop conceptual thinking basically because there is a surrounding culture that fertilizes it [32]. In this sense, word/modelling problem-based learning can be regarded as seeds in the terrain of a community of practice, since these replicate conditions that echo individuals' experiences outside the classroom, who are brought in for enculturation in a professional activity, which is at the same time the result of the work of research and communication amongst academics [33].

Accordingly, what instructors and students do in the Economics classroom is the initiation to the economic culture. Interactive communication in Economics makes possible to keep this culture 'alive', essentially because individuals are capable of reflecting upon each other's insights. So Economics learners become active members of the community by adhering to a coherent professional communication practice, in which student learners can be assimilated as novices and instructors as experts.

Do peer and/or novice-expert reflective communication necessarily imply resorting to mathematical registers, and if it does, in what extent does this happen? A likely explanation is here recovered from metacognitive mediation. Observe part C in fig.5; remember knowledge structures allow problem solving skilfulness transferal, and metacognitive mediation can be used to support epistemological understanding development. In fact, metacognitive skilfulness can also be seen as a social product [15], which in the Learning Hexagram model may translate into different levels of peer collaboration, depending on the problem's degree of difficulty:

"…metacognition tends to emerge and to be socially shared more frequently in difficult problem-solving tasks. There is no obvious need to collaborate in easy tasks-these can be rather routinely carried out. In general, previous research suggests that in order for metacognition to have an optimal effect, tasks should be located just beyond the point where a person functions proficiently independently, that is in the 'region of sensitivity' or 'Zone of Proximal Development'…" ([15], p.382)

A plausible answer to some of the question marks above then is that more sophisticated mathematical registers may tend to be used less with more difficult problems, since more verbal interaction (pre-symbolic interaction) and less written codification (symbolic interaction) might be needed. Therefore, register sophistication level could become neutral in collective economic meaning creation.

Likewise, in learning environments where challenging word/modelling problems are formulated, it would be expected that points of classroom cooperation will rise. Therefore, in Economics classrooms where evaluativist epistemological understanding is aimed, metacognitive skills are anticipated to enhance metacognitive collaboration and the other way round, which could eventually lead to collective register levels.

5. Final remarks

After scrutinizing the influence of the nature of Economic Education on the nature of Economic knowledge, some conclusions can be drawn. First, diversity of learners is accounted for when allowing for the co-existence of different mathematical registers with different sophistication levels. Second, epistemological maturity can be achieved through metacognitive questioning when using word/modelling problems because this technique induces conceptual thinking. Third, in a community of practice, collective meaning creation necessarily implies the evolution from multiplist to evaluativist epistemological understanding.

The main limitation is considered to be the actual applicability of the Learning Hexagram, since this could clash against the way instruction is delivered, for some instructors may find more 'handy' to approach concepts using mathematical registers of higher sophistication level, rather than allowing students to go 'verbally round'. Model extensions should then include task design by ability level.

Nevertheless, based on the analysis of both the semiotic and metacognitive dimensions of the Economics-Mathematics alliance, this paper has opened what are held important research avenues in Economic-related Education, particularly in the area of creative pedagogy. Under the prospect of increasingly competitive job markets and Higher Education budgetary constraints, Economics, and consequently Business, Accounting and Finance Education in Colleges and Universities, can benefit from the perspective here undertaken in the design of teaching-learning strategies.

This is true essentially because the basis of the Learning Hexagram is reality-referenced economic problems, since these are understood epistemologically through academic interaction in the search of solutions. Thereby, professional practice is simulated while enrolled in a higher education programme, enhancing so future career prospects, which constitutes a major job competitive advantage for prospective students.

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