Mathematical Capability Pre-school Children

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Abstract

Most of people have fatal attitude to Mathematics: some of them are capable to learn it form nature, but the others are not. So is their fate – to suffer from it for the whole of life… But it is a rude though natural mistake, as it results from means of mathematical education and its content. Most of parents and teachers are directed on these aspects both in kindergarten and at primary school.

Of course, parents are different. Nevertheless so many parents can't possibly but speak about achievements of their children. Some start making their own children learn better by the example of success of the others. They make their children learn long chains of figures with no understanding. It is even more sad to see how a mom asks her 4-year old son: "How much is two plus three?..' But he replies just because he learned the answer but not calculated.

Not only parents but also kindergarten tutors don't want to understand that drilling for arithmetic has ***no sense****. For a specialist it would take two days only…But teach him how to think logically – is a goal demanding from him, reached by different means.*

1. Introduction

It is clear, that capability to any subject or other activity are determined by individual psycho features, genetic predisposition. Although nowadays there is no evidence to stipulation of abilities by neural issues of any kind. Moreover, it is possible to compensate even unfavorable abilities. Task-oriented approach will lead to personal growth, formation of clear-cut abilities, which is proved by certain experience.

Mathematical abilities are from a group of so called *special abilities* (e.g. musical, painting etc.). To reveal their existence certain knowledge is needed, together with certain skills, namely skill to use knowledge in mental activity.

Mental activity – the key type of mathematical activity. Realization of its results is one of the strongest stimulations for current development of the civilization.

The problem of knowledge digestion and accumulation is traditionally connected with natural figures' apprehension and operations with them: counting, adding on, arithmetic operations and comparing, changing the scalar quantities, as well as **quantities with nonnegative results of change.**

Many educational programs create the mathematical content with the focus on "natural numbers and operations with it". The process of mathematical … formation is aimed at content (knowledge) and operational (skills) elements of curriculum. In other words, "certain knowledge base" is associated with knowing the natural numbers, whereas "collection of certain skills" can be understood as practical operations with numbers – counting, adding on and use of symbols (operational figures and signs), typical mathematical problem solutions etc.

Both Russian and foreign researchers associate formation and development of mathematical abilities among school children with mental processes (not with subject knowledge and skills).

Talented children usually have number of specific characteristics, namely, flexibility of mind, i.e. fresh thinking and ability to various cognitive problem solving, easy transfer from one problem to another, ability to come out of usual activity and find new solutions under changing conditions. Such peculiarities of mind are directly depending on specific memory organization as well as on imagination and perception.

Researchers point out also such characteristics as ***deepness of thinking***. By this they mean ability to penetrate into essence of each fact and event, observe their interconnections with other facts and events, uncover specific, implicit characteristics of the learned material.

Among major characteristics in mathematical thinking there is ***task-oriented thinking*** in combination with its ***breadth***, i.e. ability to formulate general ways of thinking, skills of team vision of a problem. Prior to all other categories mentioned above, specific or natural aptitude to structural approach to a problem and maximum stability, concentration and amount of attention. Mathematical abilities are closely connected with cognitive abilities, including *sensitive* (perception and observation of subjects and events) and *intellectual abilities* (out-coming information processing).

Consequently, task-oriented development of all mental characteristics as well as sensitive and intellectual abilities (thinking as operational process, i.e. independent analysis making, synthesis, comparison and other mental operations) on the **mathematical material** will favor general development of mathematical abilities among children.

2. Why do some challenges appear?

Special or subject knowledge allow us "speak the language of Science" – operate with sign systems? Peculiar to a particular system, reveal and describe logics of conclusions with the help of familiar symbols

(in our case – figures, letters, signs). Knowledge recorded in such a way becomes clear to an onlooker (a teacher, a tutor, parents), seeing and estimating cognitive results. Although the most important part of a mathematical process is left outside.

Initial mathematical visions of a child are formed on work with numbers and operations with them (i.e. counting and arithmetic operations). Great variety of symbols allows making the process "transparent" and controlled. On the other hand, such process can not serve development neither of mathematical thinking nor mathematical abilities.

The main way of pre-school children development is empiric generalization, i.e. generalization of their sense experience. Accumulation of such experience is based on sensory capabilities of a child (vision, hearing, sense of touch) and its "processing" is realized through intellectual capabilities. It is necessary to provide a child with conditions for investigation and experimenting, in order to start the "engine" of this process. In other words, educational content should be both acceptable by senses and favor his experimental needs. Such experimenting may result in development of a child on the way of the World perception and understanding.

You, probably, have mentioned that there is a kind of contradiction: a figure as a mathematical issue is a highly general abstraction with … from basis of its construction. Despite of the way chosen for "natural number's" construction – on the meaning of "set" or on scalar quantities' measurement – Number as the key issue of mathematics is abstract, impossible to be directly perceptible for senses. Any "object snap" of a Number (e.g. use of trees, rabbits for counting) is a double loss of abstraction and consequently loss of the essence's generality. We should speak about "double" loss because in this case we deal not with graphic image (number of pixels) but variety of trees or rabbits, etc. This image is directly percepted by a child, acts in experiments. Results are fixed in empiric generalization. This can e proved by the fact that primary school children often loose results of such generalization when the teacher change rabbits for trees, for example. They see this change as a new situation and repeat the whole process from the very beginning.

Theoretically, we may conclude about importance of numerous experiments with different objects for the sake of right empiric generalization. But in practice it is not true for many cases. Reasons are different and vary from individual perception abilities up to lack of descriptive materials. In this sense, traditional substitution of independent work with observation of the teacher's activity can not be adequate. So, contradictions mentioned contain reasons for high level of unpredictability, if we speak about creation of mathematical abilities.

Early introduction into numeric and sign symbols (i.e. early symbolization) is not widely recognized. Pre-school children learn it very easy as it is usual way of coding for their plays. Nevertheless, symbols get separate meaning due to absence of ready symbols configuration. Herewith its external manipulation replaces implicit operating with mathematical notions and relations.

There is a great variety of examples from teaching practice. They prove independence of symbols if we speak about the children's mind. At the same time, its link with real sense of notions and relations is quite peculiar. Judging from experience and examples given above it is evident to say that children can easy remember order of presentation as well as symbols themselves. On the one hand, examples show lack of flexibility and deepness of child's thinking; on the other hand, they reveal tendency to formalization (it is easier to learn strictly shaped images).

3. Arithmetic? Algebra?? Geometry!!!

There are several components in the mathematical content: arithmetic material, algebraic and geometrical materials. The first and the second ones are incorporated into quantitative characteristics of subjects and their groups (arithmetic is based on notion "number"), are connected with generalization process of their qualitative characteristics (letters are used in algebra for qualitative characteristics) and operations (algebra is based on notion "operation" equal to more general notion "actions" from arithmetic).

Even slight analysis of mathematical notions mentioned above proves that we deal with abstractions of high-level difficulty and generality. In particular, counting of apples in a set or rabbits on a meadow need a child to be disembodied from all perceived objects' qualities (color, size, shape, taste etc.). At the same time a child should concentrate on such characteristics as "quantities of variety ". As for algebraic symbols, it needs disembodiment not only from qualities and characteristics of objects but also from their quantity: x of rabbits, y of carrots.

Learning of Geometry has its specific character, too. Its major components are figures and bodies on two- and three-dimensional space. As it is possible to create models of all geometric objects, investigate and operate with them, initially and in pre-school period we usually use sensor abilities of children.

Analysis of mathematical programs and manuals for school children reveal an interesting tendency: mathematical educative material mainly consists of arithmetical material. Among typical exercises for 1st-year pupils there are counting, numbers and natural numbers' qualities accompanied with arithmetical exercises, addition and subtraction tables, arithmetical problems, multiplying and division tasks, double figures etc.

It is a kind of paradox because all notions mentioned are highly abstract and demand not on "imagination" but abilities of abstraction without sensory support, which turn to be impossible for a 5-6 years old child.

Use of geometrical content in work with pre-school children helps to omit all these methodological challenges. A model of any geometrical notion can be

directly perceived by a child. Besides there are other advantages of geometrical material use:

- It helps in work with the "Zone of proximal development" with reference to experience and knowledge of children. More difficult task motivates a child for new activities in mathematics. First, children copy models and way of work with them assisted by a teacher; then try to construct according to a picture etc.
- It helps in creation of evaluative environment with the help of new material use (but not by speeding educational process). For example, a 2-3 year child makes easy compositions operating with geometrical figures. In fact, he learns their features and qualities (sides' length, parts positions etc.) In 3-4 years age a child analyses their similarity and differences in their size, sides' length, their number etc. When a child is 6-7 years old, he compares different objects, formulates comparison's and generalization's results, makes an assessment of qualitative characteristics, describes separate space and qualitative aspects of the objects etc. In this case, we don't need annually insert new figures, enlarge list of notions, borrow new issues from school program. The only thing needed is set of new exercises, reveal of new features in familiar notions and new relations between them
- Geometrical material helps in resting on children's interest in experiments – natural means for learning the material by children of a certain age.
- It stimulates the process of mental development, necessary for any cognitive problems solution.
- It gives an opportunity for building the educational process based on plays. They are interesting for children as constructive activity itself is perceived by children as a play, makes them interesting and does not acquire additional plots.
- It promotes graduate and more stable learning the material. Initially, on the stage of an adequate mental act's formation an external base is needed. It will be used by a child as a model later on. Working with arithmetical material we may face some problems with such external basis' creation.
- By means of mathematical activity it helps in development of such qualities of a child as observance, assiduity and ability to plan succession of operations etc. So, structures of any kind need child's ability to work with notions and relations.

Effective development of mathematical way of thinking on the geometry material is connected with formation and development of cognitive abilities (both sensory and intellectual). In this context, education is based not on qualitative but space objects' characteristics. It means that first forms and motions are perceived, and then come qualities. It makes all children equal in ability to learn mathematics.

Let us formulate the main tasks of the course of child`s mathematical development:

- teaching a child available kinds of modeling and forming on its basis of the primary mathematical conceptions (number, quantity, geometrical figure and so on);
- forming and development of general methods of mental activity (classification, comparison, summarizing and so on);
- forming and development of spatial thinking;
- forming of constructive abilities and development of constructive thinking on its basis;
- forming of elementary graphical abilities and skills;
- preparation for study of maths in the primary school.

It is necessary to solve the problem of forming and development of spatial thinking because there are few major forms inside the category "mathematical thinking" (varieties of thinking), which may be characterized with special terms: spatial thinking, logical thinking, and functional thinking and so on.

Each of these forms of mathematical thinking is based in its development either on general kind of thinking, or on verbally-logical. It is clear, that spatial thinking will be based on graphical thinking, which is typical for a child of junior age, and this kind of thinking should be developed among the pre-school children.

We should notice, that using the method of modeling while teaching the mathematics because of its specific promotes the development of spatial form of thinking. But to use this method successfully it is necessary to develop child`s abilities to model the objects. On the primary levels this modeling should be mainly substantial – this kind of modeling is called constructing, as the child constructs the model that he studies of some material (for example of paper, sticks, and so on). That is why it is necessary to put a task of forming child`s constructing skills (this skills could be named practical).

The accentuation of stimulation of spatial and constructive types of thinking proves the main direction of the course to stimulate and develop graphical-acting-verbal and graphical-verbal types of thinking among the pre-school children, as the necessary level in creation of graphical and verbal-logical types of thinking during the following levels of development. The task of forming of simple graphical abilities is determined from one side by the necessity of development of modeling activity of a child (the next after the subjectial level of modeling, the child will work mainly with graphical models); and on another side by the necessity of development of small muscular system of the hand, as it is the preparation for writing. The content of the course is a list of mathematical conceptions and types of constructive actions, which fulfilling makes the child learns this conception.

Here is the list of course material of developing teaching of mathematics in the pre-school period.

Age from 3 to 5: Geometrical conceptions

The primary conception of the form of geometrical objects. Figures and objects (plain and volumetric).

Simple tasks for recognition (to choose the proper object) and comparison (to choose the proper object among the similar. Operations with geometrical figures and objects. Construction of geometrical objects of different materials. Creation of drawings of geometrical objects using the stencil. Constructing the geometrical figures of different parts (geometrical puzzles)

Circle and oval. Triangle and quadrangle. Square. Volumetric objects (sphere, cube, prism, cone, cylinder). Elements of projective examination of these figures in practical activity. Creation of compositions of spatial objects.

Orientation in space and on flatness: orientation concerning oneself and other objects. Disposition of figures and objects (under, on, in front of, behind, above and so on). Inside and outside.

Preparation to form the conception of number.

Comparison of the objects according different features with summarizing of quantitative characteristics. Quantitative correlation: many – less, one – many. Comparison of the majority of objects by establishment of accordance. Introduction to relations: bigger, smaller, equal. Choosing one, two, three objects according the principle of digital figure. Correlation of numerals with the groups of objects (one, two, three…). Introduction to quantitative and ordinal counting (till 5). The symbol of number – is a digit.

Formation of the presentations concerning the quantity.

The size of the objects. Conceptions: big – small, bigger-smaller, equal in size; high – low, higher – lower, equal in height; long – short, longer – shorter; equal in length – on the basis of comparison of two objects, which are different in one or few features.

Comparison of the objects by size: by length and mass according the sensory and kinetic sensations (visually, taking into hands), by square and volume (comparison and experimentally). Forming the conception of meanings of the features of the object. While the comparison of the features, that could be measured (length, mass, volume), use the models. Comparison of the lengths via apposition use natural measure (hand, arm, step) and conditional measure.

Formation of constructive abilities

Designing of the objects of different parts, of sticks and puzzles. Designing of the pictures and patterns of optional details. Drawing and completing of the pictures.

Age from 5 to 6 years: Geometrical conceptions

Specification of conception of form of geometrical figures: simple tasks for recognition, comparison, classification (size, shape, colour). Creation of pictures and ornamental patterns of geometrical figures, painting using the stencil. Construction of geometrical figures of separate parts (geometrical puzzles, sticks and so on).

Construction of compositions, using geometrical puzzles and sticks.

Dot. Straight line. Curve. Zigzag. Its modeling using rope, sticks and so on. Construction of straight line using a sheet of paper.

Inner and outer parts of the figure. Figure`s border. Closed and unclosed lines. Triangle. Square. Circle and circumference. Semicircle. Oval. Symmetric ornamental pattern.

Volumetric figures (sphere, cube, prism, cylinder, cone). Elements of projective analysis of those figures. Reading of simple drafts of compositions, made of volumetric figures. Creation of composition using the draft.

Preparation for the conception of number forming

Comparison of different objects with verbal description. Comparison of groups of objects. Separation of one, two or three objects from the group of objects according to given characteristics. Definitions: more – less, equal, some. Correlation: “Similar” – “Different” according to the practical exercises in comparison of the objects (equal in one characteristic, different in another). Grouping of the objects, which are equal in this or that feature, and different in other features. Understanding of the meaning of the words: each, all, the other, except.

Comparison of multitudes of objects, according to compliance: more or less, equal. Comparison by counting of elements of multitude. Different ways of equalization of multitudes.

Subjectial model of natural number. Quantitative characteristic of multitudes. Counting of objects in different directions and spatial location. Understanding of the fact, that the last numeral concerns the whole group of the objects, but not only to the last of the objects. Understanding the fact, the amount of objects in the group doesn`t depend on the size, colour, shape, distances between the objects.

Oral counting, counting of the movements. Addition and subtraction of the objects with reporting the results. Correlation of the number with the amount of the objects. Digit, as a symbol of number. Correlation of a digit with number and amount. Quantitative and ordinal counting (till 10). Ability to answer the question “which one in the count?” in a proper way. Conception of ordering by numeration of the elements (rules of counting).

Digit 0. Principe of natural number forming. Number`s place in the line of numbers. Getting numbers by adding or subtracting 1. Previous and next numbers. Comparison of numbers in different ways. Introduction into a comparison symbol. Conception of endlessness of natural numbers.

Forming of dynamic model of structure of numbers 2, 3, 4, 5.

Preparation to forming of conception of arithmetical actions.

Connection between changing of quantitative characteristics of multitude and subjectial action (changing): unification and addition leads to increasing of quantity, separation and impressments of parts leads to decreasing of quantity. Ways of equalization of groups of objects by increasing of number of objects in the smaller group, or by decreasing of number of objects in the bigger group. Commenting of the actions by words: "after addition the number of objects increased, after subtraction the number of objects decreased".

Practical actions with the objects, which can open the main point of addition and subtraction, as an introduction into the arithmetical actions.

Designation of those actions by the symbols: "+", "-". The meaning of the actions addition and subtraction. Fulfilling of those actions on the basis of subjectial model (calculating – as a way of getting the result).

Forming of conceptions of values and its dimension.

Size of the objects. Conceptions: big – small, bigger – smaller, equal by size, high – low, higher – lower, equal by height; long – short, longer – shorter, equal by length – on the basis of comparison of two (several) objects, which are different in one or several parameters.

Ways of comparison (superposition). Understanding of similarity and difference of the objects by their size. Ability to use terms for signing the size of the objects while the comparison. Compiling the groups of objects with given characteristics.

Comparison of the objects by length and mass on the basis of sensory and kinetic sensations (superposition, visually, sensory). While comparison of the characteristics, that can be measured (length, mass, volume), using the models and measuring rods.

Comparison of the length of objects, using the natural measures (step, hand). Comparison of the masses using measures: to be able to measure off "the same mass", "more", "less".

Forming of spatial conceptions.

Orientation in the surrounding space: in front of, behind, above, under, between, and so on. Comparison: higher – lower, closer, up – down, left – right, in front of – behind, inside – outside and so on, and the ability to model these relations between the objects. Orientation on the flatness of the sheet of paper. Work with volumetric forms. Plain picture of the volumetric object (frontal view) and the compositions of volumetric objects.

Forming of time conceptions.

Time, as a value, accessible for measuring. Seasons. Titles of seasons and their order. Pictorial model of seasons.

Time conceptions" today, tomorrow, yesterday. A day. Time (morning, day, evening, night). Their order. Week. Days of week.

Forming of the abilities to solve the problems on constructing.

Constructing of geometrical figures of detached parts. Creation of pictures, puzzles, models, according the given example. Creation of pictures and puzzles using the stencils. Creation of symmetric ornaments inside of different forms (inside of the circle, square and so on). Work with compasses. Excision by contour.

Three projections of prism. Constructing according the sketch. Plan. Work with the constructor according the technical task.

The main results of learning of this programme are the following skills of a child:

- To compare the objects by size, colour, form, commenting the process;
- To count different objects till 10, to answer the following questions: "how much? Which one?";
- To compare two groups of objects on basis of practical exercises, and to find out where the amount of objects is bigger, smaller, equal, to answer the corresponding questions;
- To orientate on the sheet of paper, to be able to find the top of the sheet, the bottom, the left and the right sides;
- To understand the words: between, behind, in front of, in the middle, earlier, later and so on;
- To possess the basic graphical skills: inking, hatching, drawing and so on, on the unruled paper with maintenance of spatial positions (inside, outside, and so on);
- To recognize and distinguish geometrical figures in different positions, to be able to construct of sticks and different other parts;

While building of educational programme, we can see a spiroid-dilative system. While such method of building of educational programme the sequence in studying of mathematical conceptions and relations between them is kept, but not from the point of view of linear sequence (step by step, that leads to widening of studied conceptions), but from the point of view of widening of sequence of studied relations between the conceptions. For example: While introducing the values: In the age of 3-5 years: a child studies to notice and recognize the presence of different qualities in the objects and in the groups of objects. The new knowledge of meaning of these qualities for the object is formed. A child studies to compare the objects by size: by length and by mass on the basis of sensory feelings, by square and by volume experimentally, determining, which object is heavier, bigger or smaller; and so on. While comparison of the measures a child also studies to use different models, while comparison of the qualities, that may be measured (length, mass, volume).

In the age of 5-6 years, extending the knowledge

concerning measures of the objects, a child studies to choose and to use himself optional, relative measures for measuring the length, masses of free-flowing and liquid substances. A child studies to compare the masses using different measures: to measure off "the same", "more", "less". A child studies to use natural measures (a step, a hand) while comparison of the objects, that prepares him for understanding of dual nature of natural number (a number, as a characteristic of quantity of the elements of discrete multitude, and a number as a measure).

While introduction to natural number: In the age of 3-5 years, a child studies to compare the objects by different features with gradual separation of quantitative characteristics; to compare the majority of objects using the method of establishment of mutually-monosemantic correspondence. A child becomes acquainted with relations: bigger, smaller, equal; fulfilling different actions with totalities. Учится выделять один, два, три предмета из группы по принципу числовой фигуры. A child studies to correlate the numerals with the corresponding groups of objects (one, two, three…). A child studies to separate one, two, or three objects from the whole group of objects according to the given qualities. A child becomes acquainted with the conceptions: more – less, equal, few, and so on. A child becomes acquainted with the quantitative and ordinal counts (till 5). A child becomes acquainted with the symbol of the number – a digit.

In the age of 5-6 years continues the studying of the qualities of natural numbers: a child studies to use the quantitative characteristics: more – less, equal; while the comparison of the objects using the method of establishment of bijection. A child studies different ways of equalization of multitudes.

A child studies to build the subjectial model of natural number. A child studies to count the objects in different directions and spatial locations. At the same time the understanding, that the final numeral concerns the whole group of objects, but not only the last of them is formed, and the understanding, that the general quantity of the objects in the group doesn`t depend on its size, colour, form, and the distances between the objects.

A child studies to correlate a digit, a number and a quantity. He receives the first knowledge in regulating of multitude by numerating its elements (the rules of counting). A child gets acquainted with the subjectial model of segment of natural numbers and studies to build it from different materials. Gets acquainted with digit "0", and with its place in the sequence. Gets the first knowledge about the principle of building of natural numbers. Studies to get numbers by addition or subtraction by 1. Gets acquainted with conception of the next and the prewious numbers. Studies to compare the numbers in different ways. Gets acquainted with the symbol of comparison. Gets the first understanding of unlimited natural numbers. Such building of the programme of developing education in mathematics of a child of a pre-school age corresponds with the modern conceptions of the nature and ways of building of of developing programme of subjectial education.

The principle, realized in this programme also correlates with modern points of view of developing education. According to these points of view, the methodical system in its beginning is built like an undeveloped or weakly developed structure, which later differentiates in different directions and becomes more complicated and multi-leveled. While such building of programme and education, the cognitive structures of personality, that fulfill the process of analysis of material, become more partitioned, more able to single out the details of material from the context much better; the whole slighter and slighter dominates on its parts, and a child isolates the different parts (qualities, connections) from the whole, and operates with them independently from the whole and from each other.

Such method of building of system of education of a little child will lead to the system of knowledge, that will be differentiated during the educational process, and will become in a child`s mind more developed, divided, and well-ordered cognitive structure, that will mean the achieving of the goals and receiving of the results of developing education on the basis of mathematical content.

4. Conclusion

Consequently, we may state that reasons for "mathematical abilities" being a rare case lie in educational system as such. System of introduction into the world o mathematics does not coincide with children's way of understanding it.

It is well known that not all abilities of children are seen on the surface, so a teacher needs to find, reveal them. Unfortunately, this pedagogical axiom does not work if we speak about methods of teaching mathematics. Teaching the subject is aimed at content but diminishes the key objective of any kind of education – personal development of a pupils resulting in abilities creation, mathematical abilities including.