

A Short Preparatory Calculus Course: Is it Effective?

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Abstract

In the UTSC Math & Statistics Learning Centre, we designed a 2-week summer preparatory course to help incoming students review background materials that are essential for the success in their first-year calculus courses. This paper discusses the design, implementation, and effectiveness of the course. We examined the effectiveness of the preparatory course through assessing students' mathematics skills and their confidence of their mathematics skills before and after the course. We also examined the students' perception and satisfaction with the course. The results show that students demonstrate a higher level of mastery of the fundamental mathematics skills after completing the course; they also tend to be more confident about their mathematics skills. Additionally, students rated the course as very helpful, providing them with a more positive experience than their past experience with mathematics. The course also helped the students getting used to the university learning environment.

1. Introduction

Students often have difficulties and struggle in mastering the materials in university mathematics and statistics courses. To assess and identify the reasons behind students' difficulties in learning mathematics and statistics topics and to provide appropriate supports to students, Department of Computer and Mathematical Sciences and Centre for Teaching and Learning at University of Toronto Scarborough (UTSC) collaborated on creating the Math and Stats Learning Centre (MSLC) in 2004. MSLC surveyed and interviewed students, and has identified some common challenges students face while taking university mathematics and statistics courses. Students frequently reported the courses assume a higher level of mathematics background knowledge and skills than what they have previously mastered in high school. They are also challenged by various transitional changes in the learning environment from high school to university including speed of instruction, pace of courses, and

amount of workload. They are also unfamiliar with the larger class sizes and find it difficult to interact with their peers, course instructors, and teaching assistants. These factors may also contribute to a lower confidence level in learning the material. One student expressed her frustration with her experience in university mathematics through an online survey:

"I did not realize how unprepared I was. I am really frustrated at my performance in math. I will truly be happy when I am out of the math courses. With my other courses I do not simply have the time to spend it on studying Math. In the time it takes me to study material in subject biology, say 2 hours, I would have had to study around 5 hours to absorb the same material should it have been math. I'm not saying math is impossible, my point is that math is like a crying baby... you have to constantly feed it attention or it will bother you in the future."

These concerns are also confirmed on a larger scale through data collected from a first-year calculus course offered at the University of Toronto Scarborough. 27.4% of the students in the course obtained less than 70% on a diagnostic test that was given at the beginning of the course to assess students' readiness. The data also showed that of the students who received less than 70% on the diagnostic test, 50% of them eventually failed the course. These results suggest that a significant portion of first-year students are not well-prepared for university calculus.

In order to address the concerns expressed by students, MSLC teaching staff designed and implemented different activities and services. Some of the services provided include seminars, workshops, virtual tutorials, individual appointments, small-group consultations, and a summer mathematics preparedness course. This paper focuses on the mathematics preparedness course which is designed to help promote readiness for students who will be entering the university in the coming semester. This paper discusses the design and implementation of the course, and assesses its effectiveness of the course in various aspects.

2. Mathematics Preparedness Course

The mathematics preparedness course is a two-week non-credit course, which has been offered in the summer to incoming first-year students since 2007. The purpose of the course is to better prepare students for university calculus courses. It provides intensive training in fundamental mathematics skills that are essential for the success in university calculus courses. The materials covered include algebraic manipulations, inequalities, and functions including trigonometry functions and inverse functions. The course is targeted toward students who do not have a solid background in mathematics or are not confident about their mathematics knowledge. It is intended to assist these students for a smoother transition from high school to university. It also provides an opportunity for students to get used to the learning environment and instruction style in the university.

2.1. Course structure

The course usually runs for two weeks during late summer. The paper focuses on the results of summer 2012, where three identical sections were offered to accommodate 117 students in total. This number represented 8% of the total number of students who were required to take one of the calculus courses in the following semester. Students in each section met three times per week for three two-hour lectures, and were required to complete three assignments and a final assessment. Although it was a non-credit course, students' performance were nevertheless evaluated based on their performance on the assignments and exam. In addition to their final grade, students were also given verbal comments on their progress shortly after they completed the final exam.

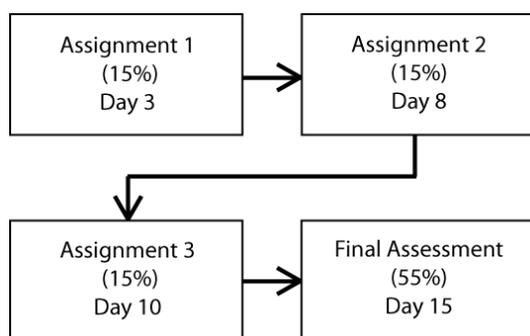


Figure 1. Timeline of the course components

Supplementing lectures, 24 hours of small-group consultation sessions were made available to students each week. These consultation sessions were run by teaching assistants. Students were encouraged to take advantage of these sessions for

help on written assignments or with questions in the course manual.

2.2. Instructional design

A growing number of educators argue that the effective approach to learning mathematics should be based on the constructivist learning model [1, 2], where students are actively involved in the learning process. In this model, instructors are considered facilitators rather than dispensers of knowledge, and students construct their own knowledge through realistic exercises and social interactions.

In the mathematics preparedness course, various collaborative learning techniques [3, 4, 5, 6] are employed to facilitate a constructivist learning environment. Interactive lectures and consultation sessions are designed to mimic mathematical thinking and communication within the discipline and to facilitate learning. Extensive small-group consultation sessions are incorporated in the course to cultivate learning through interactions between students and the teaching assistants. Furthermore, plenty of group activities are included in the lecture sessions and during small-group consultations to promote interactions between students and to foster active and thorough learning of the material.

One example of such collaborative activities is using a Three-Step Interview Technique to encourage students to exchange information, ideas, and opinions in an open and enjoyable learning environment. This technique starts with the students forming a group of three, with designated roles: interviewer, responder, and recorder. Based on a topic or question provided by the instructor, the interviewer poses a question to the responder, who provides an answer, while the recorder takes notes. Roles rotate after each 10-minute interview. The cycle is repeated until each student in the trio has been in each role (3 cycles). This is particularly useful for helping students network and improve their communication skills.

Activities such as the one mentioned above help students formulate ideas, learn to communicate clearly, and practice to be attentive and respectful listeners. In addition, students learn the concepts more deeply and will remember them for a longer period of time. The collaborative activities help students to connect what they hear and what they say to knowledge they already possess [8, 9].

Students are also strongly encouraged to be inquisitive. As it is argued in [7], "Questions help us construct knowledge. They point to holes in our memory structures and are critical for indexing the information that we attain when we develop an answer for that inquiry." Questions also provide an excellent guide for the instructor to elaborate on concepts and provide more interesting and relevant examples on the spot. Additionally, students'

confidence can be enhanced considerably when they seek the clarifications that they need.

	When do we say a function is one-to-one?	How do you decide if a function has an inverse?	What is the relation between the graphs of a function and its inverse?
Interviewer			
Responder			
Recorder			

Figure 2. An organizer for students notes for a Three-Step Interview class activity

3. Course assessment

3.1. Skills tests

To assess the improvement in students' readiness for first-year calculus courses, two skills tests were administered online; one was given at the beginning of the preparatory course and another one was made available at the end of the course after the final exam. The two skill tests were derivatives of diagnostic tests used in first-year calculus courses at UTSC. The two tests contained the same mathematical techniques but differed slightly in format (for example, in one version, students might be asked to solve for x when $3x(3x - 1) = 3x - 1$, while in the other version, students would solve for x when $x(x + 1) = 3x + 7$. These tests tapped into the fundamental skills that are essential for students' success in first-year calculus courses. They covered various topics including algebraic manipulations, inequalities, and functions including trigonometry functions and inverse functions. The tests were in a multiple-choice format, and each had 20 items.

Calculus instructors often report that in tests and exams their students make more mistakes on algebraic manipulations than the main concepts. The pre- and post- skills tests items were carefully designed based on students' common mistakes in algebraic manipulations, inequalities, and functions including trigonometry functions and inverse functions. For example Item 6 in the pre-skills test was designed to examine students' skills in solving inequalities. A common mistake in solving inequalities occurs by failing to consider the fact that

if a product or quotient has an even number of negative factors, then its value is positive. Likewise, if a product or quotient has an odd number of negative factors, then its value is negative.

Domain	Sample Pre-Skills Test Items	Related Post-Skills Test Items
Algebraic manipulation	Simplify $\frac{1}{y} + \frac{1}{x}$ $\frac{1}{y} - \frac{1}{x}$ a. -1 b. $\frac{x+y}{y-x}$ c. $\frac{x+y}{xy}$ d. $\frac{x+y}{x-y}$	Simplify $\frac{1}{y} + \frac{1}{x}$ $\frac{1}{y} - \frac{1}{x}$ a. -1 b. $\frac{x+y}{y-x}$ c. $\frac{x+y}{xy}$ d. $\frac{x+y}{x-y}$
Functions	Let $f(x) = x^2 + 1$ and $g(x) = x^2 - x$. Find $f(g(-1))$. a. 2 b. 5 c. -2 d. -5	Let $f(x) = 2x - 1$ and $g(x) = 2 - 3x$. Find $g(f(2))$. a. -9 b. 3 c. 0 d. -7
Trigonometry	If $\sin \theta = \frac{3}{5}$ then $\cos \theta$ can be $\frac{4}{5}$ a. $\frac{4}{5}$ b. $\frac{5}{4}$ and $-\frac{5}{4}$ c. $\frac{5}{4}$ only d. $\frac{4}{5}$ and $-\frac{4}{5}$	If $\sin \theta = \frac{3}{5}$ then $\cos \theta$ can be $\frac{4}{5}$ a. $\frac{4}{5}$ b. $\frac{5}{4}$ and $-\frac{5}{4}$ c. $\frac{5}{4}$ only d. $\frac{4}{5}$ and $-\frac{4}{5}$

Table 1. Sample Pre- and Post- Skills questions

3.2. Perception surveys

Supplementary to the skills tests, two surveys were implemented to assess students' perception of the preparatory course, and mathematics in general.

The surveys used a 7-point Likert scale to assess students' interest in mathematics, their past experience with mathematics, their perception of their own mathematical skills, and the impact of the course. These surveys were administered in class, in the beginning and on the last day of the course.

Pre-Perception Survey:

1. What is your level of interest in mathematics?
2. How confident are you with your mathematics skills in general?
3. How confident are you with using and reading mathematics notations and language?
4. How was your past experience with learning mathematics?

Post-Perception Survey:

1. What is your level of interest in mathematics?
2. How confident are you with your mathematics skills in general?
3. How confident are you with using and reading mathematics notations and language?
4. How was your experience with this course?
5. To what extent did the course help you getting used to the learning environment (larger class sizes, faster pace, etc) in the university?
6. How would you rate the overall usefulness of the course?

Table 2. List of the questions presented in the perception surveys

4. Results

4.1. Skill tests

Eighty and fifty students completed the online pre-skill test and post-skill test, respectively. Thirty-five students completed both tests. Figure 2 depicts the grade distribution of the tests for these students. The students' performance on the pre-skill test was 63.7%. Following the preparatory course, students demonstrated improvements in their fundamental mathematics knowledge and readiness for first-year calculus courses. Their performance on the post-skill test increased significantly to 70.1%, $t(34) = 2.035, p < 0.05$.

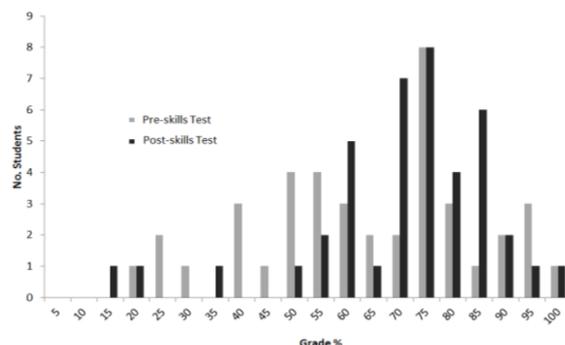


Figure 3. Pre- and post-skill tests grade distribution

4.2. Perception surveys

Seventy-eight students completed both the pre-survey and post-survey. The results of the surveys indicated a marginally higher level of confidence in students with their mathematical skills after the course ($\bar{x}_{pre} = 4.47$ vs $\bar{x}_{post} = 4.65$), $t(77) = 1.693, p = 0.094$.

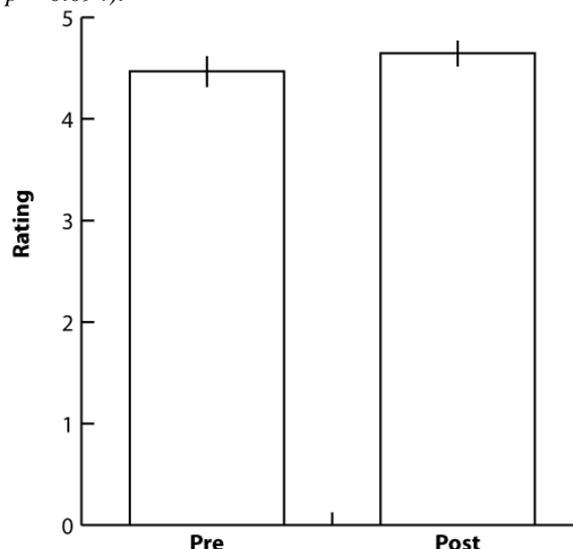


Figure 4. The improvement in confidence level

In term of students' perception of the course, the course was rated more positively compared to students' past experience with mathematics ($\bar{x}_{pre} = 3.24$ vs $\bar{x}_{post} = 4.26$), $t(77) = 4.022, p < 0.001$. Furthermore, 92.0% of the students rated the course as being helpful, and 34.7% of them said that it was very helpful.

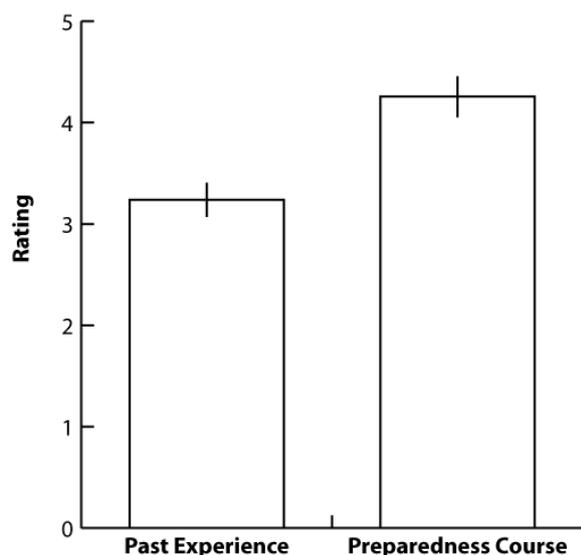


Figure 5. Improvement in students' learning experiences in mathematics

The post-survey also showed that 84% of the students expressed that the course helped them adjust to the learning environment in the university, allowing an easier transition between high school and university.

5. Conclusions

The Math & Statistics Learning Centre at the University of Toronto Scarborough offers a two-week mathematics preparedness course to facilitate incoming students' readiness for university calculus courses. In this study, the effectiveness of the course in summer 2012 was evaluated.

The course was able to target students with a weak mathematical background, and was found to have a positive impact on these students' readiness for the university calculus courses. After merely two weeks of training, students' fundamental mathematical skills improved significantly. They also became more confident in their skills following completion of the course. Hence, this summer program is proven effective in placing students in a better position for their first-year calculus courses.

In addition to improvements in students' mathematical skills, the course was also found enjoyable and was rated positively. Students also expressed that the course helped them adjust to the transition from high school to university.

Finally, the course was able to increase the chance of success in students' first-year calculus courses. A higher passing rate was observed in students who underwent the preparedness course compared to students who also had a poor mathematics foundation but did not take the preparedness course. As a follow-up, we tracked the performance of a small sample of the students in the course in one of

the first-year calculus classes offered at UTSC in the following semester. Out of the 31 students we were able to track, 24 of them (77%) eventually passed the calculus course. The passing rate is substantially higher than the 50% rate expected from students who received less than 70% on the diagnostic test.

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