Observation and Exploration of the Sky in Primary School: Spatial Thinking and Measuring Distances and Angles for developing Geometric Skills

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Abstract

Spatial thinking, a central focus of our research endeavour, is intricately connected to inherent cognitive abilities that manifest early in human development. Young children exhibit an early capacity to process and relate spatial information about objects, shapes, distances, and positions. The structured approach to astronomical observations holds significant potential for enriching our comprehension of Earth's position in the Universe and fostering the development of spatial thinking and geometry. Our educational program, tailored for 9-year-old students, integrates astronomical observations conducted through unaided visual inspection and purpose-designed instruments employing readily available materials. The activities are centred on luminous phenomena, the optics of vision, the genesis and geometric principles of shadows, the computation and measurement of remote objects, and the examination of the sky. Activities immerse the students in a pedagogical experience that empowers them to adeptly delve into concepts of plane geometry, commencing with a foundational comprehension of space.

Keywords: Spatial Thinking, Astronomy, Geometry, Primary School, Parallel Globe, Measurement, Modeling, Light and Vision

1. Introduction

Children's initial encounter with space is through their bodies. After becoming familiar with it, they assess motor skills concerning their surroundings. As a result, they use their bodies to explore the space around them. Subsequently, babies start to establish a system of internal representations essential for orientation, enabling them to employ an egocentric system to manage their position in space and an allocentric system as reference points. This intricate set of skills forms the basis of learning and is called visual-spatial skills [1]. Understanding the functions of one's body is crucial for experiencing, attributing meaning to reality, and learning [2]. Initially, the child explores one's body as the primary space they are acquainted with. With motor skills development, they also explore the surrounding space, fostering an understanding of one’s body within the environment.

Apprehending space through movement and processing spatial information is crucial for developing the capacity to form internal representations of space required for orientation. These skills encompass visual-spatial exploration, spatial perception, spatial thinking, visual-spatial working memory, and visual index-based construction skills. Spatial perception involves the three-dimensional mental representation of the environment arising from the perception of depth and movement. This process necessitates mentally manipulating representations of objects and spaces, such as rotations, translations, and perspective changes [3]. These cognitive processes are linked to visuospatial working memory, allowing the retention and processing of visual and spatial information to memorise spatial relationships between landmarks accurately. Visual-spatial skills are the foundational elements for developing mental representations essential for comprehending and navigating the world around us.

Visual-constructive and praxis-constructive skills, which are involved in developing internal representations of objects and spaces, require motor coordination, praxis organisation, and graphic skills [5]. These visuospatial skills are not just theoretical concepts but are fundamental for our effective interaction with the world around us. They aid in developing mental representations and spatial coding strategies during movement [6-7]. Mental representations are subsequently formed in the child based on spatial coordinates [8]. Recent research has demonstrated that contrary to Piaget's view, children can encode spatial information about objects, shapes, distances, positions, and spatial relationships very early. Spatial thinking, akin to numerical thinking, thus seems grounded in innate capacities that emerge early in life [9 - 14]. While visual-spatial abilities are not exclusive to humans, these abilities are augmented in humans through symbolic systems. Even in preschool age, we have a graphical language that assists in developing diverse representations of space, such as maps and diagrams, and in mentally moving and transforming objects in space. Therefore, adults must understand how to teach geometry to enable children to master those fundamental concepts often perceived as challenging progressively, thereby fostering the development of specific and more sophisticated skills based on those already present.

2. Teaching and learning of geometry

The ideas espoused by Piaget and Inhelder (1960), as delineated in their work "Child's Conception Of Geometry" [15], have significantly shaped geometry teaching. They clearly distinguished between "perceptual" space, the space perceived by the child through sensorimotor activity, and "representative" space, referring to the space a child can represent using language. Building upon this, the van Hiele couple in 1986 proposed an evolutionary theory in learning-teaching that differentiated between geometry as the conceptualisation of space and as a formal theory [16]. Their teaching model has undergone several refinements and is now foundational to numerous curricular proposals, particularly in the United States.

Emma Castelnuovo [17-18] also contributed significantly to geometry teaching by developing experiences and teaching aids that facilitate the development of the geometric intuition necessary for gradually understanding the formal geometry system. Her project allows students to define geometric entities based on corresponding natural objects and compare particular cases with borderline cases, thereby familiarising themselves with the search for the invariant elements that underpin construction in different geometries, such as that on a spherical surface.

An essential role in our experiments and research is played by studying geometric transformations with age-appropriate languages and tools, in particular by analyzing the formation of shadows in sunlight. The relationship between light phenomena and geometry has been investigated since ancient times. Euclid himself developed a hypothetical-deductive theory of direct vision in Optics. The rectilinear propagation of light and, in particular, the phenomenology of the formation of shadows offers the possibility of involving students in the construction and study of geometric models and in the use of transversal concepts, not only valuable for Astronomy. In mathematics and science, involving primary school children in recognising invariant properties in geometric transformations is significant. Geometric transformations are biunivocal correspondences between points on a plane or in space and can be classified precisely based on invariant properties. In a phenomenological approach to geometric transformations in sunlight, it is of great interest to involve children who can initially observe their shadows, standing facing the sun and in profile, with different poses and see that shapes and sizes depend on the height of the sun, the position assumed and the arrangement of the plane of the "screen" on which the shadow is projected (the floor, a wall, etc.). The invariant properties, linearity and parallelism can be studied using objects of different shapes and long threads that connect the contours of the obstacle figure and the image figure. Appropriately guided by coordinating and arranging in space obstacle figures, figures and images on an orientable flat screen, children relate the height of the Sun with shapes and dimensions and discover that isometries are particular cases of affine transformations.

The experimentation described in the following section was conceived within the scope of these influential references. Our research group has engaged with them from a phenomenological perspective for years, focusing on geometric modeling and constructing mathematical concepts and skills [19-20].

3. Teaching and learning of astronomy

Children, through experience, build beliefs concerning the physical, biological and social world; sometimes, these ideas are so deeply rooted that they persist into adulthood. In the astronomical field, it is challenging for the youngest to abandon the initial conceptions; for example, we live in the flat middle of a sphere, there is a definite up and down in space, etc. In the absence of a scientific education aimed at developing critical thinking, initial ideas may persist as adults, even in the presence of unsatisfactory interpretations [21]. In general, despite the diffusion of worthy scientific dissemination materials, the ability to argue about the accepted interpretative models is not widespread. Almost everyone knows that the Earth has a spherical shape and rotates around the Sun, but how many, to help children, know how to argue on which observations this theory is based? What are the advantages and constraints of using geocentric and heliocentric systems? To gradually build skills that are based, as always, on observations and deductions, it is good to consider the ideas that children generally show they have about our planet. Research in teaching insists on some common mental representations that present evident conflicts. Overcoming such conceptions requires careful teaching mediation that starts from children's initial ideas to guide them with observations, deductions and arguments appropriate to their age towards models that gradually appear less contradictory and plausible. The role of adults, particularly teachers, is crucial, but it appears clear that they cannot always manage the complexity of the problems they face. Lanciano has carried out extensive research on teachers' conceptions and has launched with his research group reflections and projects on how to start from the observations that children can make to build physical models and mental models that help to build theories that become plausible. The use of the parallel globe is a teaching method aimed at offering children a tool that supports them in understanding complex concepts that cannot be acquired through direct experience, such as the shape of the Earth and how the Earth's illumination changes in time and across its surface. Easy to find and use in every school, the parallel globe is oriented in space like the Earth, with the Pole Star in the direction of the globe's axis. Once positioned, the globe facilitates and promotes children's understanding of complex concepts, such as the day-night cycle and alternating seasons [22].

The parallel globe represents a prototype of the Earth in its exact position for the Sun and the other stars. The use of this instrument allows children to observe the same phenomenon in the reference system, that is, from the Earth's surface, as well as from a reference external to the Earth; furthermore, it provides them with different possibilities [23]:

- Observe the Sun in the sky and simultaneously understand through the globe where it is night at the same time.
- Perceive seasonal changes.
- Record the varying heights of the Sun in the sky throughout the year.
- Observe where the Sun "hits" in a specific month of the year.

A "normal" globe is similar to the Earth from a geometric perspective, whereas a parallel globe is homothetic to our planet, as its axis is parallel to that of the world that passes through the poles and the centre. In this regard, the horizontal plane is parallel to the tangent plane.

This type of globe has been used internationally; however, it is precisely in Spain that it has known different and new applications. Esteban, to highlight the amount of information that could be drawn from it, defines it as "bola magica", or the magic sphere. Camino Nestor, on the other hand, took care of positioning several of them in the province of Esquel in Patagonia, in southern Argentina; the photos obtained from this globe are significant to understanding how the Earth is perceived from the other side of the world [24], [25].

In the experiment described below, setting up the parallel globe was one of the most significant moments of the educational path. The whole class oriented it and identified reference systems to organise the proposed observations coherently. The concepts and operations of geometry and astronomy that involved the experience on the parallel globe could be understood as the conceptual map of the project.

4. The experimentation

The research encompassed a cohort of 22 elementary school students, 10 male and 12 female students from the fourth-grade level. The primary school teacher conducted the investigation as part of her specialised pedagogical thesis over approximately 30 hours. Preceding the activities outlined in this research, conducted during the final four months of the academic year, the educator had previously implemented the prescribed curriculum, which involved introducing the concept of angles and addressing problems related to the perimeters of plane figures. The students encountered considerable challenges in comprehending abstract geometric concepts. The experimental study, deeply practical, commenced with an emphasis on the visual process, highlighting the significance of primary and secondary sources of light and light detectors, particularly the human eye (see Figure 1).

Figure 1. The mechanism of vision, the rectilinear propagation of light, observing near and far

The students were instructed about the emission of primary radiation from light sources such as the Sun and torches and the subsequent emission of secondary radiation by objects, creating a visual perception through chains of sources.

Figure 2. Shadows, parallel rays. "The gnomon" and the motion of the Sun

The activities were designed to provide real-world applications of the concepts. They were conducted outdoors under direct sunlight, diffused light, and within the classroom using torches.

Figure 3. Look and observe how we estimate the distances and sizes of near and far objects. Reconstruction of the angle of view with a protractor made with two rods

A key aspect of the study involved observing shadows in sunlight and demonstrating the parallelism between rays and their divergence by working with point sources in the classroom.

Through measurements and estimations involving triangulation methods, the relationship between the dimensions of "obscuring" and "obscured" objects was verified. Additionally, the study identified visual angles by reconstructing angles with arms and hinged rods (see Figure 3).

The final activity involved working with a parallel world map in the schoolyard, which was oriented similarly to the Earth in space (see Figure 4). This activity allowed for real-time observations of phenomena such as day and night, the position of the Sun, and shadow direction, employing a compass and a structure to represent the terrestrial sphere.

Figure 4. The children participated in constructing the parallel world map

In Figure 4, the sphere was oriented so that the observer's position was the highest, and the sphere's axis was tilted by about 40°.

A diagram was created to represent the correlations between the topics covered and the development of practical skills, aiming to facilitate a comprehensive understanding of the educational path. These practical skills were intended to facilitate the acquisition of concepts and methods of observation and measurement, fostering confidence in students' ability to apply their knowledge effectively.

A map (see Figure 5) has been created to help grasp the correlations between the topics covered and the attention to developing experimental skills that allow the acquisition of concepts and methods of observation and measurement.

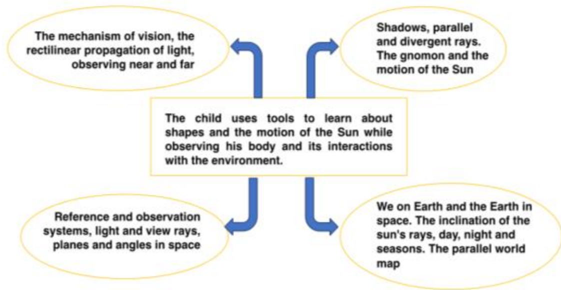

Figure 5 A map of the activities carried out in the 30 hours of experimentation

The activities are strongly interconnected with integrations between astronomy and geometry. The last activity on the parallel world map made it possible to revisit and consolidate what was done before.

5. Evaluation and Results

The present experiment was meticulously designed and implemented by the class teacher-researcher, with an intentional structure that integrates seamlessly into the established mathematics and science curricula. In alignment with the directives provided by the Italian Ministry of Education, the evaluation process was delineated into three distinct phases:

a. Ongoing Assessment. This initial phase is characterised by continuous monitoring of student progress, enabling timely feedback and adaptive modifications to instructional activities tailored to individual needs.

b. Periodic Assessment. This second phase aims to create an intermediary map of learning, illuminating any critical areas requiring targeted intervention.

c. Final Assessment. This conclusive phase involves formulating individual judgments that articulate

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the evolution of students' knowledge and skills, incorporating considerations of initial conditions and progress over time.

The assessment of learning outcomes about the experiment delineated in this study was organised according to specific indicators reflecting students' competencies associated with:

a. Spatial Perception. The ability to accurately

b. position oneself within a spatial context and estimate distances relative to one's body.

c. Spatial Language. The capacity to articulate the positions of various objects within physical space, both from an egocentric and allocentric perspective, utilising appropriate spatial vocabulary.

d. Geometric Transformations. The skill of actively visualising and manipulating spatial stimuli and executing mental translations and rotations.

e. Geometric Modeling. The ability to schematise and depict geometric entities and figures through verbal and illustrative means to assess distances.

f. Position of the Sun. Understanding the sun's position at various times throughout the day (regarding height and direction) to facilitate orientation and recognition of cardinal points.

g. Shadows and Geometry. The capacity to evaluate and identify the direction and length of shadows produced by a gnomon, alongside recognising the invariants of similar geometric transformations (concerning sunlight) and projective transformations (utilizing "point" sources in the classroom).

h. Parallel Globe. Engaging in essential experiences to comprehend the significance and application of the parallel globe.

i. Measurement Practice. Proficient use of measurement instruments, data collection in tabulated formats, evaluating quantities through direct and indirect methods, and applying proportional reasoning concerning segment lengths.

The Table 1 summarises the evaluative data for the cohort of 22 students illustrates the comprehensive assessment across the competencies mentioned above, categorising results into three levels: low, good, and excellent. Data analysis reveals that the educational intervention had a substantial positive impact across the entire class. No one scored lower than good. The school council positively acknowledged these findings, particularly about the performance of students who traditionally exhibit challenges in mathematics and science. The consensus among educators is that student success is closely linked to the motivation generated by experiential learning, mainly through outdoor activities and collaborative work in small groups and the larger classroom setting based on strong interactivity. The materials utilised for evaluation included a diverse array of evidence, such as video and audio recordings, drawings, and verbal, written, and oral descriptions reflecting the conducted experiences.

Table 1. Summary of the Evaluation Results of the 22 Students

Indicators	Good	Excellent
1. Spatial Perception	10	12
2. Spatial Language	9	13
3. Geometric Transformations	7	15
4. Geometric Modeling	8	14
5. Position of the Sun	4	18
6. Shadows and Geometry	8	14
7. Parallel Globe	7	15
8. Measurement Practice	10	12

Table 1 shows the results of the 22 students who participated in the experiment divided into the eight indicators and the two evaluation criteria. No one scored lower than good. Data demonstrate that students effectively transitioned from employing an egocentric reference system (indicators 1-3) to utilising allocentric systems (indicators 4-7). This progression emphasises the development of spatial thinking skills and familiarity with geometrical concepts that are not typically mastered at this stage of cognitive development. Moreover, the commendable outcomes observed in activities related to indicators 7-8 underscore the students' maturation in creating devices and engaging with objects, demanding an understanding of complex models related to astronomy and the adept use of measuring instruments.

The class teacher spearheaded the assessment process, grounded in specific criteria ratified by the research group. This evaluation involved all class teachers and encompassed quantitative and qualitative dimensions, focusing on analysing quantitative data to inform final assessments of each student in mathematics and science. Evaluation materials included comprehensive evidence, such as photographs, videos, audio recordings, technical and illustrative drawings, and graphs and tables developed by students, ensuring that all facets of student performance were thoroughly considered.

The evaluation framework was diligently crafted to encompass a variety of practices referring to very transversal skills and disciplinary competencies, including collaboration and coordination within groups, verbal articulation of concepts, effective communication of observational methodologies, and comprehension of fundamental astronomy and geometry principles.

6. Discussion

The geometry study in our country's schools typically begins with identifying plane figures such as triangles, squares, rectangles, and circles, then examining their properties and related problems [21]. However, due to solid geometry's perceived complexity, it is sometimes overlooked. This perpetuates a standard error in the teaching of mathematics and physics. This error lies in failing to engage students in modelling abstract entities such as points, lines, planes, geometric figures, material points, and rigid bodies in physics.

If we do not clearly explain these models, they do not facilitate understanding the relationship between real objects and abstract entities. This oversight has significant consequences, leading to a lack of appreciation for the cultural value of mathematical and physical theories, which are dogmatically present. It is crucial to recognise that everything in a child's environment has three dimensions, from their games to the furnishings in their classroom and the pen in their hand. Objects such as sticks, sheets of paper, edges, vertices of furniture, and the tip of a pencil inhabit the three-dimensional space of the classroom, either static or in motion.

We possess a rich vocabulary to describe positions, trajectories, and speed in space, and we must pay careful attention to the projections required for drawing—various experiences and research support starting with spatial concepts when teaching geometry in schools.

The process of intersecting space with planes, working with projections, and transitioning to Euclidean plane geometry is akin to the mechanism of vision. Binocular vision (see Figure 6), which relies on the slightly different images each eye receives, forms the basis of depth perception through binocular disparity. Our perception of similarity involving angles and triangles is intrinsically tied to our brain and physiology, leading to the natural imposition of Euclidean geometry on our understanding. The triangles we solve are based on the distance between the two eyes. Our binocular vision dictates the notion

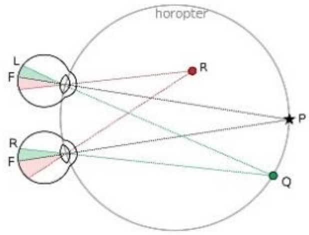

Figure 6. Binocular Vision.

of a triangle, with the short side representing the distance between the two eyes. At the same time, the brain processes the other two sides based on the distance of the observed object. From this innate understanding, we developed geometry by dividing the terrain and learned to estimate distances to distant stars using the parallax method.

Literature suggests that one of the challenges primary school children encounter in geometry is understanding the concept of angles [25], [26]. This concept is inherently complex and can be interpreted in multiple ways. The first meaning, originating from Euclid and later expounded by Proclus, defines an angle as the mutual inclination between two lines intersecting at a common point [27]. Another meaning, introduced in the eighteenth century, views an angle as part of a plane between two half-lines with the exact origin. The dynamic interpretation conceives an angle as a rotation, while another perspective sees it as a change of direction. It is important to note that focusing on a single interpretation is insufficient for meaningful learning, as each interpretation offers its own insights. While the static meaning emphasises the concept of boundlessness and the existence of two complementary angles in a plane, it alone does not capture all aspects of an angle, which can be better understood from a dynamic perspective.

One common challenge is the linguistic aspect of mathematical concepts [28]. The everyday meaning of the word "angle" (tip, edge) can lead to confusion. Consequently, it is essential to incorporate dynamic interpretations of angles and their relationship with astronomy into the teaching curriculum.

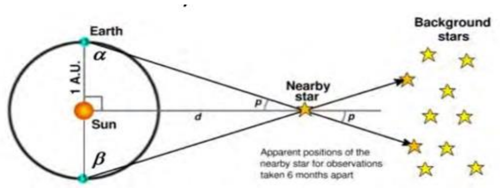

Figure 7. The Parallax Method is an "Extension" of Binocular Vision

The historical link between geometry and astronomy, particularly the critical notion of the visual angle, as elucidated in Euclid's optics, can provide valuable insights and enhance the teaching of these concepts [26]. Euclid's work is founded on the concept of visual angle, demonstrating that the apparent dimensions of an object in retreat vary according to the visual angle. This historical context can provide a solid foundation for teachers and educators to explore the relationship between geometry and astronomy with greater depth and purpose.

The study presented above sheds light on the innate nature of geometric reasoning, particularly in individuals lacking formal exposure to mathematical concepts. Pica and Dehaene's research among the Mundurucu tribe unveiled remarkable geometric abilities without explicit mathematical instruction [30].

Figure 8. Adults and Children of the Munduruc Grappling with the Concept of Angle

The study, involving adults and children from the tribe, utilised dialogues and tasks to ascertain geometric comprehension (see Figure 8). Notably, despite the absence of specific geometric terminology in their language, the Mundurucu individuals demonstrated proficiency in recognising geometric figures. Moreover, their aptitude extended to activities on a spherical surface, surpassing the performance of individuals from the Western world. This underscores the limitations of conventional pedagogical approaches to Euclidean geometry and emphasises the value of embodied cognition in learning.

Figure 9 shows how we start from the visual angle, and using tools for a dynamic definition of the angle, we then get to work with the protractor in the study of plane geometry.

Figure 9. Visual Angle using tools for a Dynamic Definition of the Angle

Finally, by centering the learning process on bodily experiences and sensorimotor activities, it is possible to advance comprehension and overcome challenges in geometry teaching, laying the groundwork for understanding fundamental astronomical concepts.

7. Conclusion

Children need to acquire a unified vision of scientific phenomena by learning to use their bodies, artefacts, and measuring instruments, as well as targeted strategies to share methods of investigation and interpretative models.

The experience we described was based on a strong integration between topics covered in geography, astronomy and physics with the possibility of building crucial mathematics concepts, particularly geometry. The development of visual-motor coordination is based on visual information from the environment-shaped geometric entities. It proved to be of fundamental importance to build and share meanings related to abstract concepts whose usefulness was manifested in the activity of prediction and measurement. Among these, the angle with the "rays of view" and the support of wires and long sticks allowed us to estimate and evaluate distances of objects even very far away. The educational path developed continuously up to the construction and work with the parallel globe, indicating that by specifically strengthening the skills related to spatial orientation, it is possible to deal with even complex phenomena thanks to the gradual control and development of geometry concepts. The outdoor activity for direct observation also involved children who were usually less motivated in studying and showed greater interest in study activities in class. The authors are happy to interact with educators, researchers and teachers to provide insights and share similar experiences.

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