Modelling Effects of Rehabilitation and Relapse in a Host Community with Social Issues

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Abstract

Social issues affect education, thereby creating instability and harm in the society. In this study, we present a mathematical model of the effect of rehabilitation and relapse in host communities with social issues such as poverty, depression, and crime. This model incorporates randomness to reflect the inherent uncertainties in these complex social issues. We developed a system of differential equations to represent the transitions between different states of individuals within the community. Analysis of the behavior of the model to understand how changes in rehabilitation programs, poverty alleviation efforts, and mental health interventions can influence relapse rates, depression levels, and crime within the community. The model provides insights and recommendation for policymakers and community leaders regarding resource allocation for rehabilitation programs, poverty reduction initiatives, and mental health services.

Keywords: Mathematical models, rehabilitation, relapse, poverty, depression, crime.

1. Introduction

Prolonged periods of remission and relapses among substance-abusing individuals, poverty, and depression are among the socioeconomic and psychological characteristics that are frequently linked to high crime rates in different regions. In many places, poverty plays a major role as a catalyst for crime. Social assistance, work prospects, and education are frequently inaccessible to those who live in poverty [1]. The interplay between poverty, depression, and crime is a complex and persistent global issue in education and contemporary society. Poverty often creates an environment where individuals face financial instability, lack of access to education, and limited opportunities, all of which can contribute to mental health problems such as depression. Depression, in turn, can drive individuals to make desperate decisions, leading to criminal behavior as a means of coping or survival. High crime rates are frequently associated with socioeconomic factors such as poverty, substance abuse, and untreated mental health issues, which perpetuate cycles of relapse and criminal activity [2] Poverty often leads to limited access to education, jobs, and social assistance, driving some individuals toward crime for survival. Understanding this cycle is crucial for developing effective policies and interventions that address not only the economic root causes but also the mental health and social stability issues that stem from poverty [3]. The mental health of individuals is shaped by the social, environmental, and economic conditions in which they are born, grow, work, and age. Poverty and deprivation are key determinants of children's social and behavioral development [4].

Studies have shown that communities with high poverty levels experience elevated crime rates, as the lack of social support exacerbates both mental health crises and criminal activities [5], [6]. In a study Chibaya and Nyabadza [7] examined the relationship between poverty and HIV/AIDS, demonstrating that impoverished populations are more vulnerable to health crises, which further destabilize communities. There are studies on depression, which links mental health challenges among youth to underlying socio-economic issues, particularly poverty. In Scotland, individuals in the most deprived areas report higher rates of mental illness and suicide than those in affluent areas, with early-life inequalities worsening over time. For example, 14.7% of 7-year-olds in Glasgow's most deprived areas display behavioral difficulties, compared to 3.6% in the least deprived [8], [9]. Understanding the life-course impact of poverty on mental health is also important. Childhood adversity is one mechanism through which poverty and deprivation have an impact on mental health. Adverse childhood experiences, such as exposure to abuse or household dysfunction, are relatively common in the population. The prevalence of seven adverse childhood experiences among children born in 2004–2005 in Scotland, and found that approximately two-thirds had experienced at least one adverse experience by age 8 [10]. Moreover, the prevalence was greatest in low-income households: only 1% of children in the highest-income households had four or more adverse childhood experiences, compared with 10.8% in the lowest-income households. Adverse childhood experiences are also strong predictors of mental health in adulthood: individuals who have experienced at least four are at a considerably greater risk of mental ill health, problematic alcohol use and drug misuse [11]. It has also been suggested that experiences of childhood adversity and complex trauma may contribute to Glasgow's – and Scotland's –excess mortality, particularly that which is attributable to violence, suicide, and alcohol and drug-related deaths. In areas with high rates of crime, the cycle of rehabilitation and relapse is a serious problem.

Many drug-abusing individuals who resort to criminal activity enroll in recovery programs, but upon their release, they relapse. Stigmatization, inadequate support networks, and limited access to ongoing care are major factors that contribute to this cycle. The cycle of treatment and relapse not only has an impact on the individuals involved but also puts a heavy load on law enforcement and community resources [12]. The focal reason for restoration is that discipline can forestall future wrongdoing by improving the singular guilty party's way of behaving. Recovery might include schooling and professional projects, guiding, intercession projects, or abilities preparation.

Previous studies have utilized deterministic models to explore the interactions between poverty, depression, and crime. However, these models often fail to account for the inherent randomness and unpredictability in social systems. Deterministic models yield a specific outcome, while stochastic models incorporate randomness, as in stock market predictions. Dynamic models track changes over time and are widely used in fields like epidemiology and engineering [13], [14], [15]. This work delves into the critical connection between depression, a significant mental health concern, and criminal activity. Understanding this link is crucial for developing effective interventions to reduce crime and improve public safety. The study incorporates poverty as a factor influencing rehabilitation outcomes. mathematical model can be used to identify the most effective allocation of resources for rehabilitation programs, considering factors like poverty, and depression. The findings can inform policy decisions aimed at reducing crime rates by addressing the root causes, such as poverty and mental health issues. This study uses a compartmental mathematical model with a stochastic approach to analyze these dynamics. We use compartmental stochastic model to analyze the complex interactions between poverty, depression, use rime. This approach allows for the inclusion of random fluctuations, providing a more comprehensive understanding of these social phenomena.

A stochastic mathematical model focusing on the effects of rehabilitation and relapse in a community dealing with depression, poverty, and crime is essential for comprehensively understanding and addressing these interconnected challenges. By capturing the inherent variability and unpredictability of human behavior and societal dynamics, such a model can provide valuable insights for policymakers, researchers, and practitioners striving to foster sustainable improvements in community well-being.

2. Mathematical Model

A mathematical model is a theoretical depiction of a framework utilizing numerical language. This language can incorporate factors, constants, conditions, imbalances, and other mathematical items. The model catches the fundamental components and connections inside the framework, permitting us to numerically investigate and control it. Mathematical modeling is the application of mathematics to describe real-world problems and investigate important questions that arise from it. Using mathematical tools, the real-world problem is translated into a mathematical problem, which mimics the real-world problem

Mathematical modeling is a powerful tool used to represent real-world systems using mathematical structures. It allows us to understand complex processes, predict future behavior, and make informed decisions. Numerical models can take many structures, including dynamical frameworks, factual models, differential conditions, or game hypothetical models.

Mathematical modeling provides significant benefits; it enables predictive analysis for planning and risk assessment, abstracts complex systems into simpler forms for analytical study, allows hypothesis testing through simulations, supports optimization of functions and resources, and facilitates clear communication through visual representations like graphs and simulations, enhancing collaborative understanding. Mathematical modeling faces limitations, such as necessary simplifications and assumptions that may reduce accuracy. Accurate data is crucial, as low-quality data may result in skewed outcomes. Complex models can be challenging to interpret, validate, and verify, making them less accessible. Mathematical models are of various types: Deterministic Models, Stochastic Models, Static Models and Dynamic Models. Determistic models have a single, well-defined outcome for a given set of inputs. Examples include models for calculating the trajectory of a projectile or predicting population growth without random factors.

Stochastic Models incorporate randomness to represent variability in the system. They might use probability distributions to predict the likelihood of various outcomes. For instance, a model for stock market fluctuations or disease spread would likely be stochastic. Static Models represent a system at a specific point in time, offering a snapshot of its state. They are useful for analyzing a system's current behavior. These models provide a snapshot at a particular time, without considering the temporal changes (e.g., linear programming models for optimization). A dynamic model is a mathematical or computational representation of a system that changes over time. It captures how variables in the system evolve, interact, and influence each other dynamically. Unlike static models, which provide a snapshot of a system at a particular point in time, dynamic models account for the effects of past states and future projections, making them useful for understanding and predicting how systems behave over time [13].

Moreover, models used in healthcare or policy have ethical implications, as they can impact decisions and public perceptions [16], [17].

3. Model Formulation

We design and analyze a non-linear model for poverty, depression, and crime by dividing the whole population into five different compartments. Interactions Between The flow between these compartments is governed by several interaction rates, each corresponding to a transition or influence. These transitions can be interpreted through a system of differential equations representing the changes over time. The flow from one state to the other is given by is given Figure 1.

Let H_p denotes humans susceptible to poverty, depression, and crime, P_v Humans in poverty D_p, Humans in depression $\mathrm{C_h}$, Humans in crime and $\mathrm{R_h}$ Humans rehabilitate. Table 1 gives the description of other parameters.

Table 1. Description Parameters

S/N	Parameter	Description
1	ϕ	The recruitment rate
2	β_1 and β_2	The influence rate
3	α_1 and α_2	Progression to crime
4	$\varepsilon_1, \varepsilon_2$ and, ε_3	The relapses rate
5	μ	The mortality rate/natural death rate
6	σ_1 *and* σ_2	Progression to rehabilitee/recovery
7	δ	The death due to crime acts
8	W_i	Is the wiener's process. where (i=1-5)

These transitions can be interpreted through a system of differential equations representing the changes over time given by Equation (1)

$$\left.\begin{aligned} \dot{H}_p &= \phi - \beta_1 H_p P_v - \beta_2 H_p D_p - \mu H_p + \gamma_{H_p} H_p dw_1 \\ \dot{P}_v &= \beta_1 H_p P_v - (\alpha_1 + \sigma_1 + \mu) P_v + \varepsilon_1 P_v + \gamma_{P_v} P_v dw_2 \\ \dot{D}_p &= \beta_2 H_p P_v - (\alpha_2 + \sigma_2 + \mu) D_p + \varepsilon_2 D_v + \gamma_{D_p} D_p dw_3 \\ \dot{C}_h &= \alpha_1 P_v - \alpha_2 D_p + (\mu + \delta) C_h + \varepsilon_3 C_h + \gamma_{C_h} C_h dw_4 \\ \dot{R}_h &= \sigma_1 P_v - \sigma_2 D_v + \mu R_h - \varepsilon_1 P_v - \varepsilon_2 D_p - \varepsilon_3 C_h + \gamma_{R_h} R_h dw_5 \end{aligned}\right\} \quad (1)$$

The Euler-Maruyama method which is a numerical method can be used to solve this stochastic differential equations[18], [19], [20].

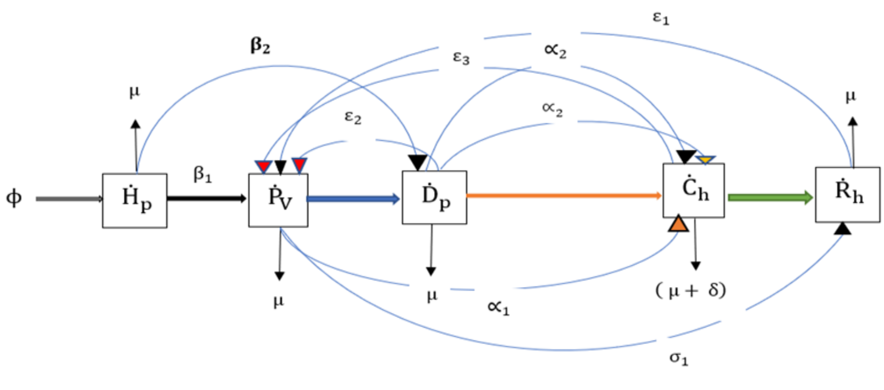

Figure 1. Model Flow Diagram

4. Numerical Simulations

The deterministic model provides a smooth trend, while the stochastic model captures real-world variability. The stochastic model is more realistic, highlighting the importance of considering random fluctuations in planning and interventions. Both models tend to stabilize over time, but the stochasticmodel shows persistent variability. Stochastic curves exhibit peaks and troughs, indicating periods of higher and lower states within each compartment. The fluctuation in the stochastic model highlights the requirement for versatile and responsive approaches to oversee unforeseen changes. The curves in Figure 2 indicate a gradual decrease in poverty levels, with stochastic fluctuations. Economic policies are having a positive impact. However, the variability indicates that economic conditions are sensitive to external factors. Poverty can lead to feelings of worthlessness, frustration, and chronic stress, which can further exacerbate mental health issues. Financial counseling, access to social services, and empowerment programs can help individuals manage and overcome poverty. Psychological support is essential to address the emotional impact of financial hardship. Likewise in similar trend to poverty, there is decline in humans with depression but noticeable stochastic fluctuations. Mental health interventions are effective but need to account for the unpredictable nature of individual responses and external stressors.

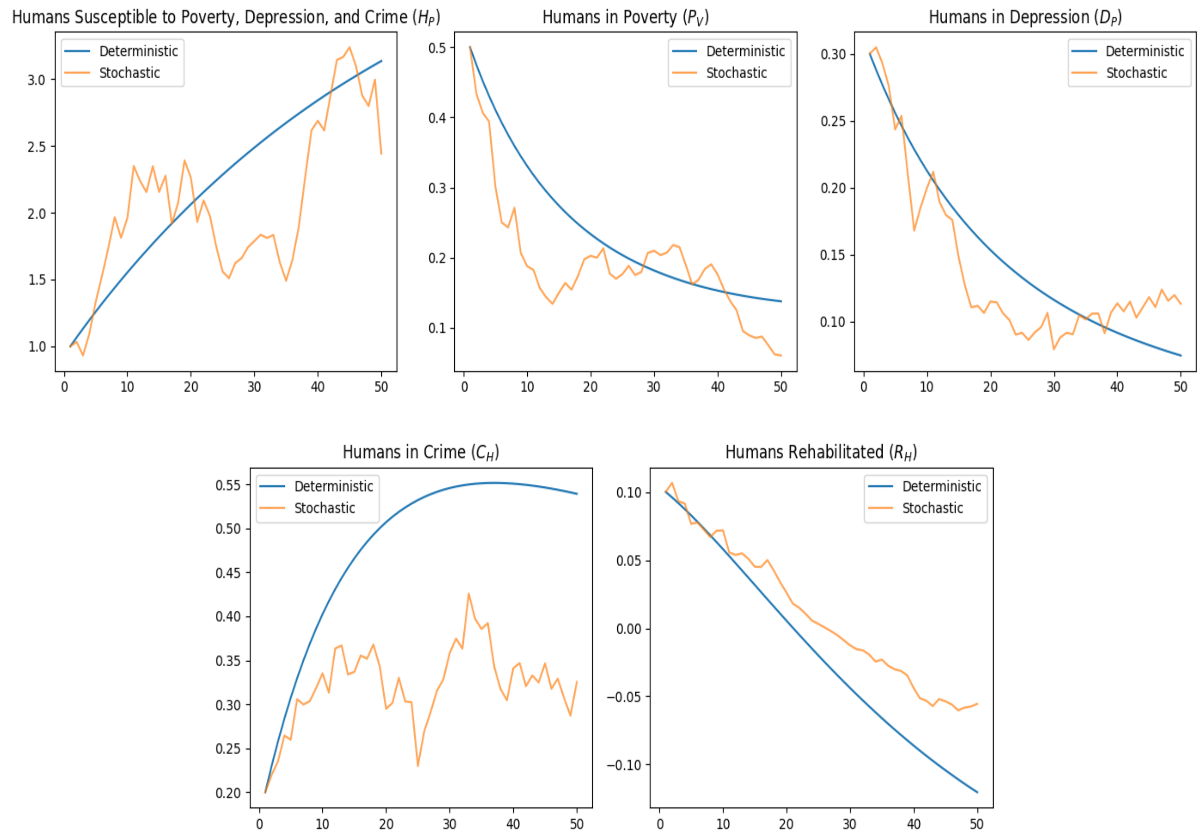

Figure 2. Behavior of the model compartmental variables in the deterministic and stochastic senses

Depression affects an individual's ability to function daily, leading to social withdrawal, reduced productivity, and a diminished quality of life. Access to mental health care, including therapy and medication, is crucial. Encouraging social connections and activities that promote mental well-being can also help. Rehabilitation numbers increase over time, with stochastic deviations.

Rehabilitation programs are effective, but the variability suggests that post-rehabilitation support is crucial to maintain gains. Successful rehabilitation can lead to improved self-esteem, better social relationships, and a more positive outlook on life. Continuous support, access to education and employment opportunities, and mental health services are vital for maintaining rehabilitation gains.

In Figure 3 the effect of β_1 on P_v, is seen as β_1 increases, the deterministic and stochastic curves show different behaviors. A higher β_1 typically increases the influence rate, leading to higher poverty levels initially, followed by a quicker decline as interventions take effect. Increasing β_1 suggests stronger social influences on poverty. Policies need to address these influences directly, such as through community support and economic development programs. Effect of on reveals that higher values indicate more significant psychological stress on individuals in poverty due to stronger social influences.

5. Conclusion

The model captures the dynamics of human populations susceptible to poverty, depression, and crime using a compartmental approach. We develop a system of differential equations or a wiener process to represent the transitions between different states of individuals within the community (e.g., in rehabilitation, relapsed, depressed, employed, engaged in criminal activity). The deterministic and stochastic models provided insights into how different parameters affect the levels of poverty,

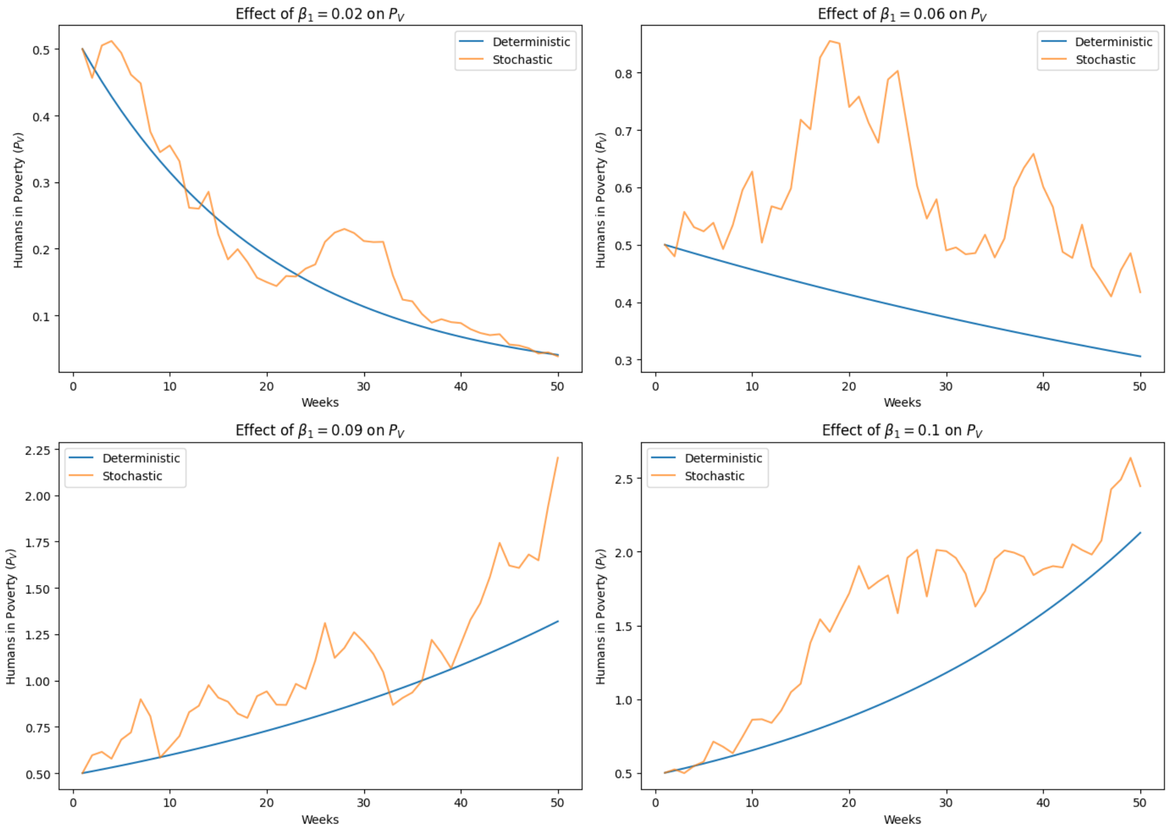

Figure 3. The effects of varying β_1 on humans with poverty fixing other parameters

depression, and crime. The deterministic model offered a baseline understanding, while the stochastic model accounted for real-world randomness and unpredictability.

It was noted that higher influence rates lead to increased levels of poverty and depression, respectively, with significant initial rises followed by gradual declines as interventions take effect. Increased relapse rates significantly affect the population dynamics, indicating the necessity of continuous support to prevent relapses into poverty, depression, and crime. It is therefore clear that implementation of robust economic policies are needed that address the root causes of poverty, such as job creation, fair wages, and affordable housing. Continuous financial support and education programs can help mitigate the influence rates. Expansion of access to mental health services to address depression, early intervention and continuous support can reduce progression rates to crime and manage relapse rates. To be effective, intervention strategies must be all-encompassing, addressing the underlying reasons for substance misuse while offering sustained support for recovery. A comprehensive strategy that addresses mental health disorders, poverty, and the cycles of relapse and rehabilitation is necessary to address high crime rates in communities. To break the cycle of crime and promote safer, healthier communities, policies that strive to reduce economic inequality, improve access to mental health treatments, and offer ongoing support to people in recovery are imperative.

It is recommended that development of community support programs are to be encouraged to strengthen social networks and resilience. Local area policing and commitment can assist with diminishing crime percentages and keep changes from destitution and sorrow to wrongdoing. Introduction of enhanced rehabilitation programs for individuals recovering from poverty, depression, and crime and continuous monitoring and support are essential to maintain gains and prevent relapses.

Regularly review and adjustment of policies based on ongoing data collection and analysis is very essential. Future work can expand on this model by incorporating additional factors such as education, employment status, and access to healthcare.

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Acknowledgements

The authors wish to acknowledge generous support from the Tunji Olowolafe Foundation and Ekiti State University, Ado-Ekiti, Nigeria.