Designing Tool-based Task in the Teaching of School Mathematics

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Abstract

This paper presents the core ideas of an on-going research exploring teaching approaches for school mathematics with mathematics teachers designing teaching tasks making use of concrete tools and manipulatives. The theories of instrumental genesis and semiotic mediation are used as the main frameworks for the study. A pilot study prior to this on-going research is reported highlighting three tool-based task design principles that are used a guide for the on-going research.

1. Introduction

This current research[1] aims to explore teaching approaches for school mathematics with mathematics teachers designing teaching tasks making use of concrete tools and manipulatives. Traditional mathematics classroom is usually conducted in a deductive and authoritative manner where teacher presents known abstract mathematical facts and symbols to students, and students use them to solve problems. Students are usually not motivated to realize that mathematics can be experienced through the use of concrete objects relevant to their prior knowledge and experience. Mathematical experiences can be mediated through tools and tools are commonly considered as resources susceptible of enhancing teaching and learning. For examples, wooden unit blocks or abacus-like tool helps children develop arithmetic, and interactive dynamic ICT platforms enhance students' understanding of geometry. Teachers play a complex and critical role to exploit the pedagogical potentials of tools. Thus a starting point for research should focus on how teachers design tool-based teaching and learning tasks. This study aims to explore and develop tool-based mathematics teaching and learning task design models that are practical for teachers to use and serve as frames for further theoretical development. The main research questions are:

1. How school mathematics teachers design teaching tasks using tools in the teaching of school mathematics? 2. How teachers integrate different tool environments (for example, ICT environment) into mathematics classrooms in designing tool-based teaching tasks for specific mathematic topics? 3. How to develop a model of task design using tools for school mathematics which teachers can use practically in the classroom?

2. Theoretical Frames

The purpose of this study is to explore and develop a practical and theoretical approach in designing teaching and learning tasks where mathematics can be experienced and learnt through the use of concrete real objects. A mathematical experience can be seen as "the discernment of invariant pattern concerning numbers and/or shapes and the re-production or re-presentation of that pattern." ([1], p.2) Using tools to empower students to discern and experience mathematics is a viable channel to expand the space of inquiry for students and to transform the mathematics classroom into a quasi-laboratory for experimentation.

A structural use of concrete objects (manipulatives) in mathematics teaching and learning was developed by Zoltan Dienes early in the 70's [2]. For example, Dienes designed the Multibase Arithmetic Blocks to provide children with multiple embodiments of the structure of our place-value number system. Uttal et al [3] offered a perspective on the use of concrete object to teach mathematics where manipulatives are considered as symbols and teachers intend for them to stand for or represent a concept or written symbol. A US research study explored problem solving in elementary classrooms focusing on how children use (perform tasks) manipulatives (or tools) in problem solving while working on mathematical tasks concluded that "if manipulative use becomes an integral part of the academic structure for all students in mathematics classrooms, it may keep more students in higher-level math classes through college and beyond." [4].

The use of tools in mathematics classroom has been a common practice for teachers; however, there is yet a common agreed overarching approach on how to transform a concrete tool into a pedagogical instrument in the mathematics classroom. An artefact

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in a mathematics classroom can be any tool that has the potential to mediate a piece of mathematical knowledge. A familiar example would be a pair of compasses which embodied the concept of circle and distance. With properly designed teaching tasks, students can experiment with it to understand the mathematics of circle. The relationship between artefacts and knowledge is very complex. In this connection, Rabardel [5] proposed a theory of instrumental genesis to explicate how the usage of a tool can be turned into a cognitive instrumentation process for knowledge acquisition. Mariotti and Bartolini [6] have been developing the notion of tool of semiotic mediation for the mathematics classroom under which a tool takes on multiple pedagogical functions. On the one hand, personal meanings are related to the use of the tool while students accomplishing a task; on the other hand, mathematical meanings are related to the tool and its use. The dual relationships constitute the semiotic potential of a tool. An artefact is regarded as a tool of semiotic mediation if it is intentionally used by the teacher to mediate a mathematical content through designed didactical intervention. Thus teachers play a critical role in the process of tool mediation. In fact, the teacher is part of the mediation process. Figure 1 represents the complex classroom dynamic of the tool mediation process [7]. The dynamism consists of a network of interactions in which the teacher through the tool (artefact) acts as a mediator between the mathematics knowledge domain and the student activity task domain.

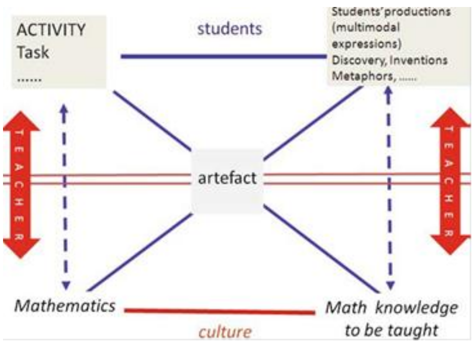

Figure 1. The Tool of Semiotic Mediation Network

Mariotti experimented with a group of Italian teachers implementing this framework using an ICT tool for geometry [8]. In the teaching experiment, teachers designed and implemented a sequence of classroom tasks using the ICT tool and orchestrated the classroom into a collective knowledge inquiry environment. The findings confirmed that the frame offered by the tool of semiotic mediation in a mathematics classroom serves as a means to support teacher's didactical action on the one hand, while on the other hand to promote students to a collective inquiry learning mode through accomplishing tasks. In particular, the researchers were able to develop a description of how the teacher can design a semiotic task process based on the use of a tool and to outline the schemes that may be put in place to promote the evolution of mathematical knowledge during collective classroom discussion.

In the tool of semiotic mediation framework depicted in Figure 1, activity tasks play a critical role in the semiotic process. The author is currently developing a pedagogical framework for tool-based mathematics task design that has the potential to be merged into the instrumental genesis and the tool of semiotic mediation frameworks [1]. This design framework consists of three nested epistemic modes of practices, discernment and discourse (see Figure 2) which characterize three types of task design orientation.

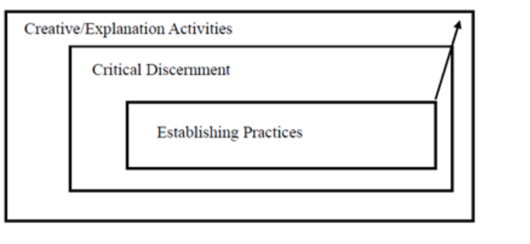

Figure 2. A nested pedagogical frame for tool-based task design

These nested modes can be seen as forming an instrumental and semiotic process in acquiring mathematical knowledge using tool. The upward arrow in Figure 2 can be interpreted as a progressive transformation of an artefact from a concrete external object to a personal internal psychology tool during a well-designed mathematics exploration task.

Combining these theoretical frameworks and realizing the critical role teacher plays in the tool mediation process, this research begins the investigation on how teachers design and implement teaching tasks in their mathematics classrooms based on this combined tool-based pedagogical model.

3. Methodology

This is an exploratory research on the formation of pedagogical practices. 16 to 20 Hong Kong mathematics school teachers' pedagogical practices of designing classroom teaching tasks, implementing the tasks and refining the tasks will be studied. Three Hong Kong primary schools and five secondary schools have been invited to participate in this project to ensure having broad perspectives on the teaching style, the choice of mathematics topics, and the choice of tools. The selection is based on teachers' willingness and commitment to participate, the academic and administrative support from school management, and the existence of collaborative culture. These are important factors to ensure meaningful data. To track the pedagogical formation process and due to the small number of the subject population, using qualitative research approach is more viable to capture the critical elements of the development. In particular, the main sources of data will be generated from research lessons and teacher clinical interviews.

3.1. Research Lesson

A series of workshops are conducted for the participating teachers from the schools, one at the beginning of the project, one after the completion of research lessons, and one at the end of the project for sharing and dissemination. The aim of the first workshop is to introduce the project to the teachers and to initiate the school-based research lessons. Each teacher conducts school-based research lessons exploring task design practices using appropriately chosen tools for specific mathematics topics. The tools in this study concern with mathematical potentiality, a wide variety of tool (in particular, ICT tools will be included) are accepted. To make the study more focused and the data more meaningful in terms of data comparison and model building, teachers are suggested to choose or design their own tools that fall into two types: Tools for measurement and geometrical construction and tools to develop arithmetic and algebraic senses.

3.2. Clinical Interview

Clinical interview has been used in mathematics education research to probe into the mind of learners (either teachers or students) about their perceptions of phenomena concerning the teaching and learning of mathematics. Selected participating teachers are invited for a longitudinal sequence of clinical interviews at different stages of the project to help construct different development personal profiles. These profiles will form important data to form a basis for the formation of the tool-based task design model, which is the aim of the project.

3.3 Data Analysis

Each research lesson is treated as a research by itself. In particular, the development of a teacher's task design for the chosen mathematics topic taught under the chosen tool environment is the focus of this level of analysis. A meta-level study across the research lessons is conducted to find overarching patterns that contribute to the development of tool-based task design model. For the research lessons, coding system and rubric will be devised and developed to analyses teachers' task designs. For each research lesson, a lesson profile on teacher and student activities will be created based on the coding system. These lesson profiles will be put together for contrast and comparison.

4. Significance

The findings of this research will contribute to a research gap where different existing approaches (theoretical and practical) of tool usage in designing mathematics classroom activities meet but there is yet a more structured framework to tie them together. In particular, it is expected that mathematics tool-based task design heuristics and principles will be developed in this research that will be relevant to inquiry-based pedagogy, formative assessment, and other educational practices that involved the use of tools. This research will serve as a stepping stone to study students' tool-based learning trajectories, and in particular, how experiential learning and theoretical learning can be bridged. Technology is developing at an exponential rate, and the nature of reality is becoming less easy to define. Integrating advanced technological tools into the mathematics classroom challenges the nature of mathematical knowledge, and in particular, the question of what represents mathematics. In this connection, tool-based task design principles and heuristics evolve with emerging technology. The findings in this research will be relevant to this important educational trend.

5. A Pilot Study

5.1. Background

A pilot study [2] was conducted prior to this research studying two groups of Hong Kong primary school mathematics teachers' pedagogical practices of designing, implementing, refining and evaluating lessons on the mathematical topic symmetry involving the use of tools. Two local Hong Kong primary schools (School A and School B) participated in the project.

In School A six Primary 4 (Grade 4) teachers were involved to collaboratively design a lesson teaching line symmetry. One of the teachers designed and constructed a "symmetrical ruler"[3] out of transparency plastic sheet. The design of the ruler was based on the idea of putting together two identical transparent rulers one on top of the other and flips the top ruler along the short edges. This flipped ruler thus has a vertical line of reflection/symmetry (the short edge) and a horizontal centimeter scale in the middles with half of the numbers in the scale as the reflected images of the other half. Transparent plastic sheets were used to make this special ruler and right angle signs were placed in all the corners (see Figure 3).

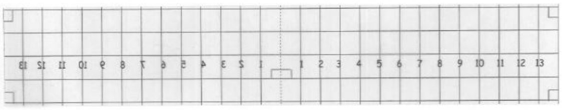

Figure 3. A teacher-designed symmetry ruler constructed out of transparency plastic sheet

This ruler was used as the main tool for the research lesson. The implemented lessons were peer-observed. Each teacher observed the lesson before his/her own lesson implementation (this needed a re-

[2] This work was supported by Hong Kong Baptist University Faculty Research Grant 11314030.

[3] The symmetrical ruler patent was successfully registered in Hong Kong

scheduling of the teaching time-table) in order to learn from it and to refine his/her own lesson. Each teacher was interviewed after his/her lesson to discuss and reflect on the implemented lesson. All six teachers implemented the lesson using the symmetrical ruler with the same basic lesson plan but with different interpretations.

In School B six Primary 5 teachers were involved to collaboratively design a lesson teaching rotational symmetry. The teachers designed and constructed a "transparency toolkit" out of transparency plastic sheet and plastic shapes. This toolkit was used as the main tool for the research lesson except for one teacher who designed a digital version of the transparency toolkit using Powpoint software for her lesson.

Figure 4. The transparency toolkit consisted of blue-tack, push pins, transparent plastic sheet, plastic geometric shapes like square and triangle, a pen and a Styrofoam board. It was designed for the purpose to verify whether a given/constructed figure has rotational symmetry

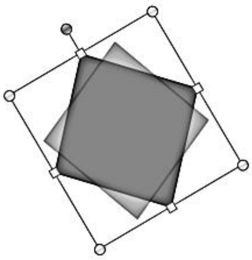

Figure 5. This is a snapshot of the digital version of transparency toolkit. The 'rotational control handle' in Powerpoint enables the students to rotate a shape or a group of shapes. While rotating the shape or a group of shapes using the 'rotational control handle', the original figure and the rotated image co-exist on the screen

As in School A, the implemented lessons were peer-observed. Each teacher observed the lesson before his/her own implementation of the lesson in order to learn from it and to refine his/her own lesson. Five teachers implemented the lesson using the toolkit with the same lesson plan but with slight variation. One teacher taught the lesson using the digital tool with a different lesson plan. A final post lesson conference was conducted involving all the participating teachers to discuss and reflect on their implemented lessons.

Overall, the teachers from both schools were innovative in designing and constructing their own tools to teach the chosen symmetry topics. The tools used were immersed nicely into the lessons creating inquiry pedagogical environments. Teachers were struggling at first but becoming more comfortable later in the whole process of learning how to focus on using a tool to teach mathematics. Students were happily engaged in "playing" with the tools. What students learnt depended very much on the teachers' pedagogical content knowledge of mathematics.

5.2. Findings

In School A, the symmetrical ruler was designed by a teacher (Teacher A below) who tried to embed all the critical aspects of the formal mathematical definition of line symmetry into the tool. However for the other teachers, given them the ruler, they had different interpretations of what this tool could do for them. In the implemented lesson, Teacher A, the designer of the symmetrical ruler, was keen to make sure students learn the mathematical meaning of line symmetry using the ruler. She was consciously guiding students to focus on equal distance and perpendicularity while using the ruler. Students were led by Teacher A to reach the desired conclusion and they learnt how to use the SR as designed by Teacher A. In this lesson, Teacher 1 played a major mediation role leading students' tool-based explorations converged to the same direction. In contrast, Teacher B began the lesson by a paper folding activity to consolidate students' prior knowledge about line symmetry. Almost half of the lesson was spent on this activity before he introduced the symmetrical ruler to the students. Teacher B allowed students to explore freely how to use the ruler in different ways and did not paid special focus on perpendicularity and equal distance. In contrast to Teacher A's lesson, students used the ruler's square grids more often as a measuring tool and folded the ruler along the dotted line to verify symmetry as fold-and-match. Not all students in Teacher B's lesson grasped the concepts of perpendicular bisector (the core teaching point of the lesson and the intended design focus of the symmetrical ruler), but students had different ways to interpret line symmetry using the ruler and the ruler was used more as an interesting drawing tool to facilitate creative thinking than the intended conceptual tool pointing to a mathematical definition.

In School B, a physical tool and a digital tool were designed embedded with the same mathematical concept that was intended as the object of learning for the research lesson. For the physical tool, a teacher was able to capitalize on an inaccuracy produced by the tool (the hand-drawn image on the transparency did not match exactly the concrete figure below under rotation) to open up a mathematical discussion with the students leading to a mathematical concept (how to find the rotation angle) that went beyond the lesson objective. For the digital lesson, students were expecting perfect accuracy from the digital tool, but the 'rotational control handle' in Powerpoint uses a different interpretation of the centre of rotation than the intended mathematical interpretation, consequently the students were struggling with making their grouped figures to rotate accurately, usually without success. The teacher at the end had to do a "chalk talk" to consolidate the lesson.

These two observations showed that while utilizing a tool for teaching and learning, the didactical interactions among the tool, the teachers and the students may generate feedbacks and discrepancies that could create opportunities or pitfalls in the pedagogical process.

1. The designed intention of a tool may not be exactly the same as the designed pedagogical intention. A tool can be "customized" by a teacher to achieve his/her didactical goal.
2. A tool has an *instrumental distance* (c.f. [9]) from the intended mathematical concept to be taught. That is, there is limitation and constraint in the tool to represent "perfectly" the intended mathematical concepts. Teachers can make use of this discrepancy to create cognitive conflict for student to engage in critical thinking and explorative activities.
3. Teacher's sensitivity to identify tool discrepancy opportunity/pitfall and ability to adjust the tool's instrumental distance are critical pedagogical skills to successfully bring about the semiotic mediation potential of tool-based task (c.f. [10]).

These three preliminary findings from the pilot study are used as guiding tool-based task design principles for the main on-going research.

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